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Temporal Aggregation and Seasonal Adjustment

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Temporal Aggregation and Seasonal Adjustment

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Analytical Services Branch

Methodology Advisory Committee

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INQUIRIES

The ABS welcomes comments on the research presented in this paper.
For further information, please contact Dr Mark Zhang, Analytical Services Branch on Canberra (02) 6252 5132 or email <analytical.services@abs.gov.au>.

TEMPORAL AGGREGATION AND SEASONAL ADJUSTMENT

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QUESTIONS FOR THE COMMITTEE

1. Do you believe the discussion presented in this paper will be a valuable reference for users to understand why the ABS might change its seasonal adjustment practice if such a decision is made?
2. Is the temporally derived approach for seasonally adjusting quarterly series a reasonable alternative method?
3. Are the revisional impact assessment measures/methods presented in this paper appropriate to the topic of interest?
4. Are the quality assessment measures/methods presented in this paper appropriate to the topic of interest?
5. Is it a concern that the temporally derived approach is less smooth for the initial seasonal adjusted estimates?

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TEMPORAL AGGREGATION AND SEASONAL ADJUSTMENT

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ABSTRACT

Suppose a time series of quarterly seasonally adjusted estimates is desired from a quarterly original time series that is the temporal aggregate of a monthly original time series. These estimates can be obtained via two approaches. Either by (1) seasonally adjusting the quarterly original time series directly, or by (2) seasonally adjusting the monthly original time series and then temporally aggregating to the quarterly frequency. For the Census X11 method, the literature suggests that seasonal adjustment first and temporal aggregation second is the more efficient approach.

Monthly original time series contain richer information for non-regular calendar related effects and outliers than their quarterly temporal aggregates. To abstract from this we have considered the impact of temporal aggregation on regular seasonal calendar-related effects only. We then determine whether there are any disadvantages to applying approach (2) to produce seasonally adjusted estimates.

Quarterly seasonally adjusted estimates for the two approaches were produced with both the ABS X11 concurrent method and the ABS X11 with ARIMA forecasting extension method. It was found that for these ABS X11 methods, approach (2) is as efficient as approach (1), results in no worsening of current end revisions and improves consistency. On balance, considering all quality dimensions, the temporally derived approach appears the superior seasonal adjustment method for equivalent monthly and quarterly time series pairs.

1. INTRODUCTION

Due to user demand and National Accounts requirements, the Australian Bureau of Statistics (ABS), in some instances, publishes original, seasonally adjusted and trend estimates at different observation frequencies for the same indicator variable. Hence at the quarterly frequency, sometimes original time series estimates are simply a temporal aggregate of their monthly counterparts.

Suppose a time series of quarterly seasonally adjusted estimates is desired from such an equivalent time series pair. These estimates can be obtained via two approaches. Either by (1) seasonally adjusting the quarterly original time series directly, or by (2) seasonally adjusting the monthly original time series and then temporally aggregating to the quarterly level (referred to as the temporally derived approach in short hereafter).

The ABS currently uses approach (1) for seasonally adjusting equivalent time series pairs because there is a perception that aggregation first improves time series stability and direct seasonal adjustment is more accurate and smooth. However, the monthly time series is richer, information-wise, since the number of observations is three times larger than for the quarterly time series. The loss of information due to aggregation before seasonal adjustment causes quality and consistency issues between equivalent seasonally adjusted monthly and quarterly estimates along with duplicate work.

Approach (2), the temporally derived approach, is of interest to the ABS because it could, potentially:

1. improve the quality of relevant quarterly seasonally adjusted estimates by better estimation (and removal) of calendar-related effects;
2. increase the consistency between the seasonally adjusted estimates of equivalent time series pairs; and
3. improve productivity of the ABS's seasonal adjustment analysis.

Driver 1: Calendar-related effects and prior correction

The estimation and removal of outliers and calendar-related effects (seasonal adjustment) is more accurate for monthly time series than quarterlies. For example, Zhang *et al.* (2005) demonstrate this for trading day effect estimation. Seasonal adjustment methods for quarterly time series that use monthly equivalents are hence expected to produce seasonally adjusted estimates of a higher quality. The temporally derived approach is such a method.

Driver 2: Consistency between monthly and quarterly estimates

There are often consistency issues between seasonally adjusted monthly and quarterly time series estimates for equivalent pairs. These differences can be due to the different application and estimation of trading day and other calendar-related effects for monthly and quarterly estimates, inconsistent application of prior corrections (large extremes, trend breaks, seasonal breaks, moving holidays, survey effects, etc.) and/or the application of other differing X11 settings (e.g. seasonal moving averages). Based on the monthly seasonally adjusted series, the temporally derived approach provides 'complete' consistency between seasonally adjusted equivalent time series pairs, and avoids the potential risk of misinterpretation and confusion by users.

Driver 3: Improvement of productivity

The temporally derived approach can improve productivity by simply aggregating monthly seasonal adjusted estimates into quarterly frequency. This leads to associated reductions in analysis and processing, and removes any potential perceived inconsistency between equivalent monthly and quarterly seasonally adjusted estimates and associated client and/or user queries.

This study focuses on comparing the two seasonal adjustment approaches, limiting the scope to applying X11 seasonal adjustment decomposition to time series with <u>regular seasonal effects only</u>, referred to as 'clean' time series. This is because we know from our previous research and literature that trading day, moving holiday, other non-regular calendar effects, and outliers can be estimated more accurately from monthly time series than from their temporal aggregates. 'Clean' time series are those which do not contain outliers and/or any 'non-regular' calendar effects such as trading day, moving holiday etc., or those that have had such effects removed.

In summary, we are interested in determining whether the X11 seasonal adjustment decomposition of clean series favours the derived temporal approach or the direct approach. In this paper, original time series are 'clean' time series as described above unless we specify differently.

This paper presents an empirical quality comparison of the quarterly seasonally adjusted estimates obtained via the two approaches, using the ABS X11/ARIMA forecasting method, in terms of their relative efficiency, revisability and consistency. The aim of this paper is to test whether the temporally derived approach is more efficient, results in no worsening of current end revisions and improves consistency.

This paper is structured as follows. In Section 2, we describe the main results and conclusions in terms of revision performance, relative efficiency and consistency. Section 3 provides a review of temporal aggregation issues in the literature. Section 4 explores approaches (1) and (2) from a seasonal adjustment filtering perspective, and relates this to relative revisions. In Section 5 our evaluation methods are explained. Results from an empirical evaluation using Monte-Carlo simulations and real data are presented and discussed in Section 6. Some concluding remarks and possible directions for future work are then presented in Section 7.

2. MAIN RESULTS AND CONCLUSIONS

Given that outliers, trading day, moving holiday, and other non-regular calendar effects can be estimated more accurately from monthly time series than from their temporally aggregated quarterly time series, the scope of this study was limited to exploring the X11 seasonal adjustment decomposition property of the two approaches.

From the perspectives of seasonal adjustment filter properties and relative revisions, the two different approaches are different, particularly for the asymmetric seasonal adjustment filters. However, these differences do not provide sufficient information to determine which approach is superior.

Our simulation results against the stable seasonal adjusted estimates suggest that the temporally derived approach improves robustness performance, measured by its reduced revision size, compared to direct adjustment. The improved robustness performance is especially strong for recent estimates, using the ABS X11 method with and without the ARIMA forecasting extension. The temporally derived approach also causes no unacceptable loss in the quality of the seasonal adjustment.

From the analysis of nine randomly selected real time series from the ABS Balance of Payments, the seasonal adjustment quality of the temporally derived approach is deemed as good as the direct approach in terms of smoothness and residual seasonality. However, an examination of reduction in revision size is not conclusive.

By the nature of the temporally derived approach, consistency between monthly and quarterly seasonally adjusted equivalent estimates is ensured, along with seasonally adjusted additivity.

On balance, considering all quality dimensions, the temporally derived approach appears the superior seasonal adjustment method for equivalent monthly and quarterly time series pairs.

3. LITERATURE REVIEW

The seasonal adjustment and temporal aggregation problem has attracted attention from academic researchers and official statistical agencies for a long time. Their interests have ranged across the effects of temporal aggregation information loss on the unit root of the time series in trend analysis, causality analysis, accuracy and reliability of seasonal adjustment and forecast performance.

Assuming the final purpose of seasonal adjustment is the extraction of the non-seasonal components and that the loss function to be minimised in so doing is the mean square error, Geweke (1978) claimed that optimal seasonal adjustment followed by temporal aggregation provides results of substantially higher quality for quarterly seasonal adjustment in terms of mean square error.

Wei (1978) studied the effects of temporal aggregation on the ARIMA model structure, parameter estimation and forecasting. He concluded that temporal aggregation will complicate the model structure, cause a tremendous loss in parameter estimation efficiency, and can cause a substantial loss in forecast efficiency.

Lee and Wei (1979) showed that for forecasting under the Census X11 procedure, it is more efficient to use the monthly seasonally adjusted series (then to temporally aggregate) than the quarterly seasonally adjusted series for forecast purposes. Based on an ARIMA model, the forecast from the monthly frequency is more favourable especially when lead time is small. These findings are most relevant to the ABS's seasonal adjustment practice because the ARIMA forecasting extension method is used to reduce seasonally adjusted estimates revisions at the current end of a time series.

Lee's (1982) dissertation claims that, for the Census X11 procedure, seasonal adjustment of monthly time series should always precede temporal aggregation to the quarterly frequency. He shows this by calculating the efficiency measure, mean square error (MSE), using the known true seasonal adjustment 'benchmark'. However, in practice the true seasonal adjustment benchmark is not known.

Di Palma and Savio (2001) explored the revision property from an ARIMA model based decomposition perspective (as developed for TRAMO/SEATS by Gomez & Maravall, 1997) for the question of interest. They found that temporally aggregating the monthly time series to quarterly time series, then seasonally adjusting, would be preferred if the objective is to minimise revision error. They claim that the order of seasonal adjustment and temporal aggregation depends on the ARIMA model and parameter range of the monthly time series.

Burgess (2007) stated that the Bank of England has adopted the temporally derived approach as new policy for its quarterly seasonal adjustment practices. Burgess cites the following 'drivers' for the change: reduced user confusion due to seasonal adjustment non-additivity, consistency with other central banks in line with international best practice, reduced production costs and analysis resource savings. He also added that the temporally derived seasonally adjusted quarterly series may reveal movements which the directly adjusted series does not, and suggested that the estimates derived from information in the monthly series are superior.

No literature explicitly explores temporally derived seasonally adjusted estimates and associated revisions under the Census X11 style method (the ABS X11 method is one of them) with and without the ARIMA forecasting extension, as far as we know. The revisional impact of the temporally derived approach for the ABS X11 method with and without the ARIMA forecasting extension is the gap required to be explored to see if the temporally derived approach provides gains in reliability for quarterly seasonal adjustment. Therefore, revision analysis under the ABS seasonal adjustment method is the main objective for this paper along with other seasonal adjustment quality measures and assessments described in Section 5.

4. SEASONAL ADJUSTMENT FILTER AND TEMPORAL AGGREGATION

Let $\{y_t\}$ be a flow random variable observed at monthly frequency. Seasonally adjusting $\{y_t\}$ can be regarded as applying a linear filter $\psi(B,F)$ where ψ is a polynomial function of the backshift operator B and forwardshift operator $F = B^{-1}$. That is, the seasonally adjusted estimator of y_t is $\psi(B,F)\{y_t\}$. Apart from some special calendar-related effect adjustment, such as adjustment for trading day, and Easter effects etc., Wallis (1982) and Dagum (1996) showed that the iterative X11 seasonal adjustment can be approximated by applying a suitable symmetric linear filter (i.e. $\psi(B,F) = \psi(F,B)$) for middle observations and a set of asymmetric linear filters for observations close to the ends of a time series.

The combination of applying the 3×5 (3×5) seasonal and 13 (5)-term Henderson filters is a common X11 seasonal adjustment setting for monthly (quarterly) time series. Let $\psi_m(B,F)$ and $\psi_q(B,F)$ denote the above seasonal adjustment filters for monthly and quarterly time series respectively.

We define $\{Y_T^*\}$ as the quarterly temporal aggregate of $\{y_t\}$. $\{Y_T^*\}$ is assumed to be available for only every third period (1×3, 2×3, 3×3, ...) where 3 is the quarterly aggregation frequency, and T is the quarterly time unit. The temporal aggregation process can be seen as two processes (1) $Y_t = (1+B+B^2)\,y_t$ and (2) $Y_T^* = Y_{3T}$ samples Y_t every 3-months. Therefore,

$$Y_T^* = W^*(B)\,y_t \tag{4.1}$$

where $W(B) = 1+B+B^2$, and * is the 3-month sampling operator.

The two seasonal adjustment approaches for the quarterly flow series $\{Y_T^*\}$ can be regarded as applying a combination of filters to the monthly flow series $\{y_t\}$ as described below.

Approach 1 (Direct – directly seasonally adjusted quarterly estimates):

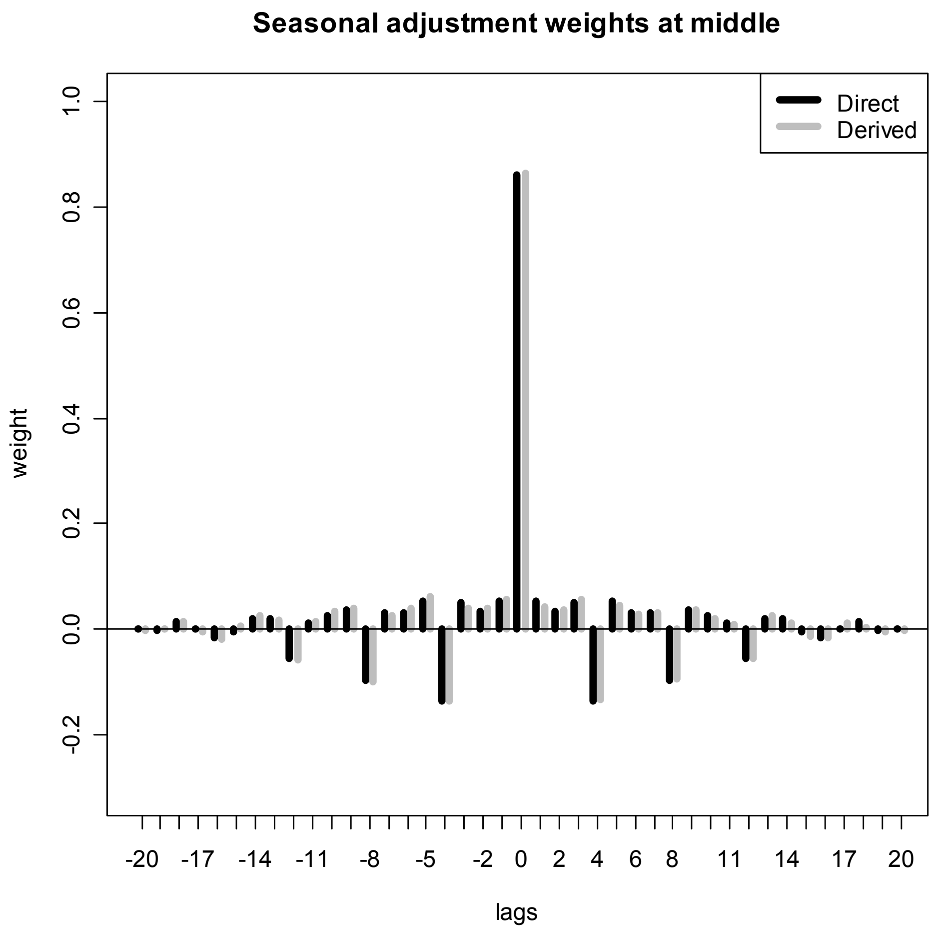
$$\psi_q(B,F)Y_T^* = \psi_q(B,F)W^*(B)\,y_t \tag{4.2}$$

Approach 2 (Derived – temporally aggregated monthly seasonally adjusted estimates):

$$W^*(B)\psi_m(B,F)\,y_t \tag{4.3}$$

Figure 4.1 depicts the seasonal adjustment weighting patterns of the direct and derived approaches for the middle part (using symmetric filter weights) of the quarterly series. Figure 4.1 shows that the two seasonal adjustment approaches yield slightly different weighting patterns although they are very similar. Therefore we expect the resultant quarterly seasonally adjusted estimates to be very similar for the middle part of a quarterly time series.

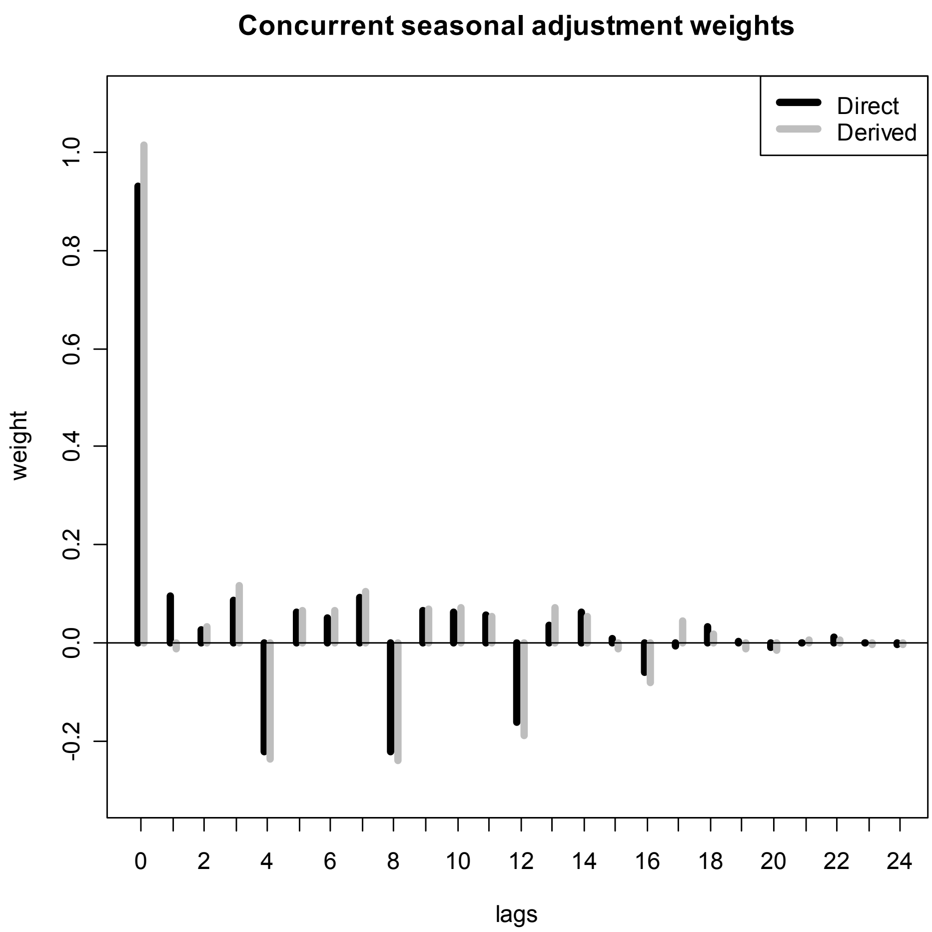
4.1 Weighting patterns for direct and derived seasonal adjustment approaches for middle section (using symmetric filter weights)



The above analysis explores the weighting patterns of seasonal adjustment for the middle observations where the symmetric weights are utilised. We now explore what happens to the current end observation where an asymmetric weight is applied. This is of particular interest in understanding the impact of the two approaches on revisions.

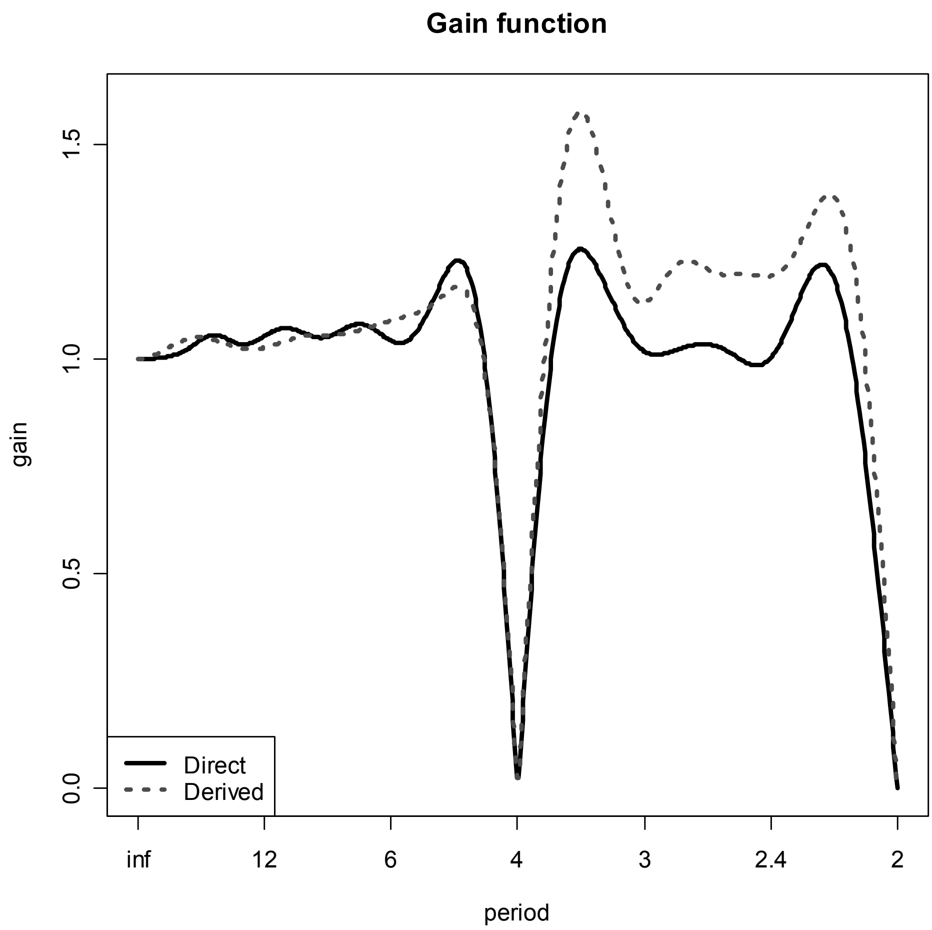
Seasonal adjustment revision can be attributed to the forecasting errors caused by the asymmetric seasonal adjustment filter applied when future observations are not available at a certain time point. Figure 4.2 shows the concurrent seasonal adjustment weighting patterns of the two approaches at the current end. Again we note that the weighting patterns are different, particularly at lags 0, 1 and 17, but not greatly. Comparing figures 4.1 and 4.2, we can easily see that the differences between the concurrent asymmetric weighting patterns of the two approaches are relatively larger than those between the symmetric weighting patterns. Hence we expect the resultant initial concurrent seasonally adjusted estimates to reflect these differences, and possible different revision properties. Notably, the revision becomes negligible after 20 or so lags (quarters).

4.2 Weighting patterns for direct and derived seasonal adjustment approaches for current end (using asymmetric filter weights)



To further understand the impact of the concurrent seasonal adjustment weighting patterns applied to the current end observation under the two seasonal adjustment approaches, we plot their gain functions in figure 4.3.

4.3 The concurrent seasonal adjustment gain functions of the two approaches



The gain functions in figure 4.3 suggest that from a quarterly seasonal adjustment perspective the derived approach performs equally well or even better than the direct approach at mid and low frequency ranges. However, the gain functions also reveal that the derived approach amplifies the power more than the direct approach at the frequencies corresponding to between two and four quarters. This indicates that the concurrent derived seasonal adjusted estimate would be less smooth than the direct seasonally adjusted estimate. This volatility is not usually observable in practice because initial concurrent estimates will be revised by later consecutive estimates after more data become available.

To further understand revisional behaviour at the current end, we next investigate the X11 asymmetric seasonal adjustment filters applied when another observation becomes available at the current end under the two approaches. We refer to these as 'the second asymmetric weights' from here on.

4.1 Relative revision properties and simulation study

Suppose $\{y_t\}$ is a monthly flow time series. When the latest observation y_t is observed, the X11 style seasonally adjusted estimate for y_t is obtained by applying the concurrent (lag 0) asymmetric seasonal adjustment weights to $(\ldots, y_{t-1}, y_t)$. This is actually equivalent to applying the symmetric seasonal adjustment weights to $(\ldots, y_{t-1}, y_t, \hat{y}_{t+1|t}, \ldots)$ centred at t where $(\hat{y}_{t+1|t}, \ldots)$ are the implied forecasts, embedded in the concurrent asymmetric seasonal adjustment weights, of $(y_{t+1}, \ldots)$ given $(\ldots, y_{t-1}, y_t)$. When y_{t+1} is available, the second (lag 1) asymmetric seasonal adjustment weights are applied to $(\ldots, y_{t-1}, y_t, y_{t+1})$. If $y_{t+1} - \hat{y}_{t+1|t} \neq 0$, revisions to the seasonally adjusted estimates will be observed at t.Similarly revision will be observed at $t-k$ when the $k+2$th (lag $k+1$) asymmetric filter is applied. We define the revision against the previous estimates described above as relative revision in this context.

We now concentrate on exploring this relative revision, particularly for the second seasonally adjusted estimate when an additional data point is available.

Let $\{z_{t+1}\} = (\ldots, y_{t-1}, y_t, y_{t+1}) - (\ldots, y_{t-1}, y_t, y_{t+1|t}) = (\ldots, 0, 0, y_{t+1} - \hat{y}_{t+1|t})$. The impact of additional data y_{t+1} to the previous seasonally adjusted estimate at t can be assessed by applying the second (lag 1) asymmetric seasonal adjustment filter to $\{z_{t+1}\}$. It can clearly be seen that the revisions are determined by the product of the second (lag 1) asymmetric seasonal adjustment weights and $y_{t+1} - \hat{y}_{t+1|t}$.

To understand the revision properties of the two different seasonal adjustment approaches (see equations (4.2) and (4.3)) for a quarterly time series which is the temporal aggregate of its monthly observations, we conducted a Monte-Carlo simulation with 1000 replicates as follows. A 'standardised' monthly time series $\{z_{t+3}\} = (\ldots, 0, 0, z_{t+1}, z_{t+2}, z_{t+3})$ was constructed where $(z_{t+1}, z_{t+2}, z_{t+3})$ are independently identically distributed (i.i.d.) as N(0,1), to represent the assumption that the forecast errors of $(y_{t+1}, y_{t+2}, y_{t+3})$ are i.i.d. N(0,1). The related quarterly time series $\{Z^*_{T+1}\}$ was then produced by applying the temporal aggregation and quarterly sampling operator $W^*(B)$ (see equation (4.1)) to $\{z_{t+3}\}$.

Approach 1 (Direct) revisions are,

$$\psi_q^{(1)}(B,F) Z_T^* = \psi_q^{(1)}(B,F) W^*(B) z_t$$

where $\psi_q^{(1)}(B,F)$ is the second (lag 1) quarterly asymmetric seasonal adjustment filter.

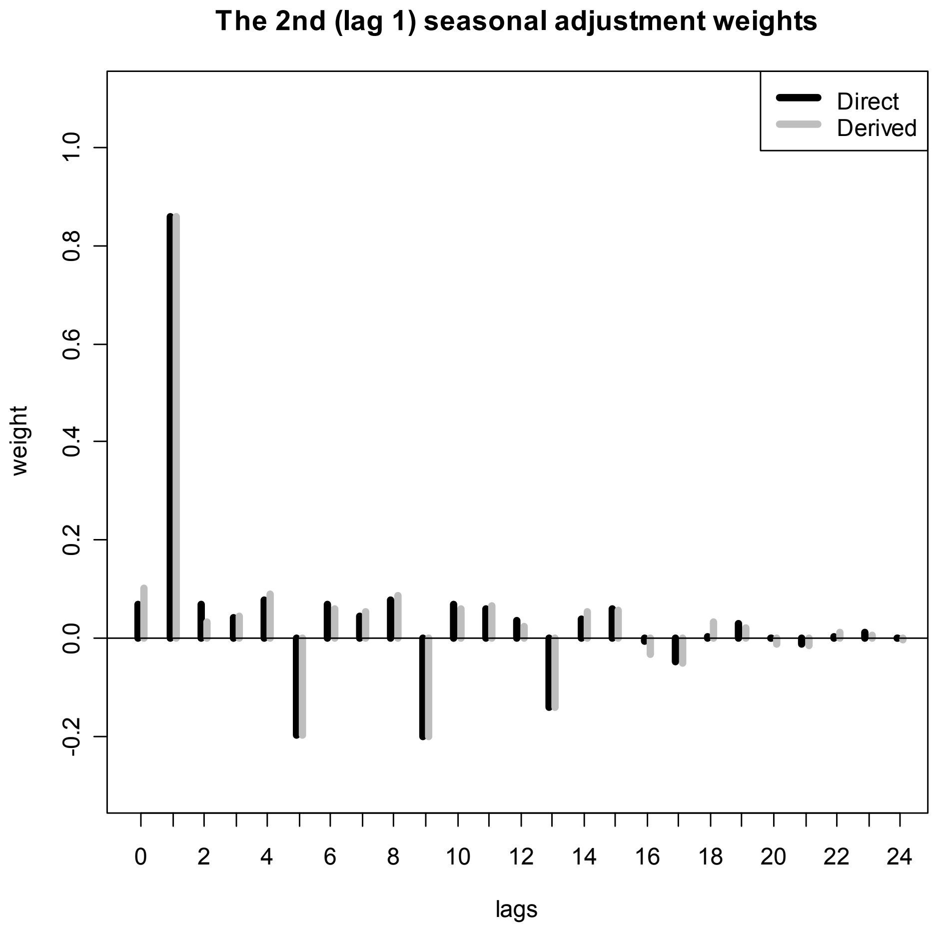
Approach 2 (Temporally Derived) revisions are,

$$W^{*}(B)\psi_m^{(3)}(B,F)z_t$$

where $\psi_m^{(3)}(B,F)$ is the fourth (lag 3) monthly asymmetric seasonal adjustment filter.

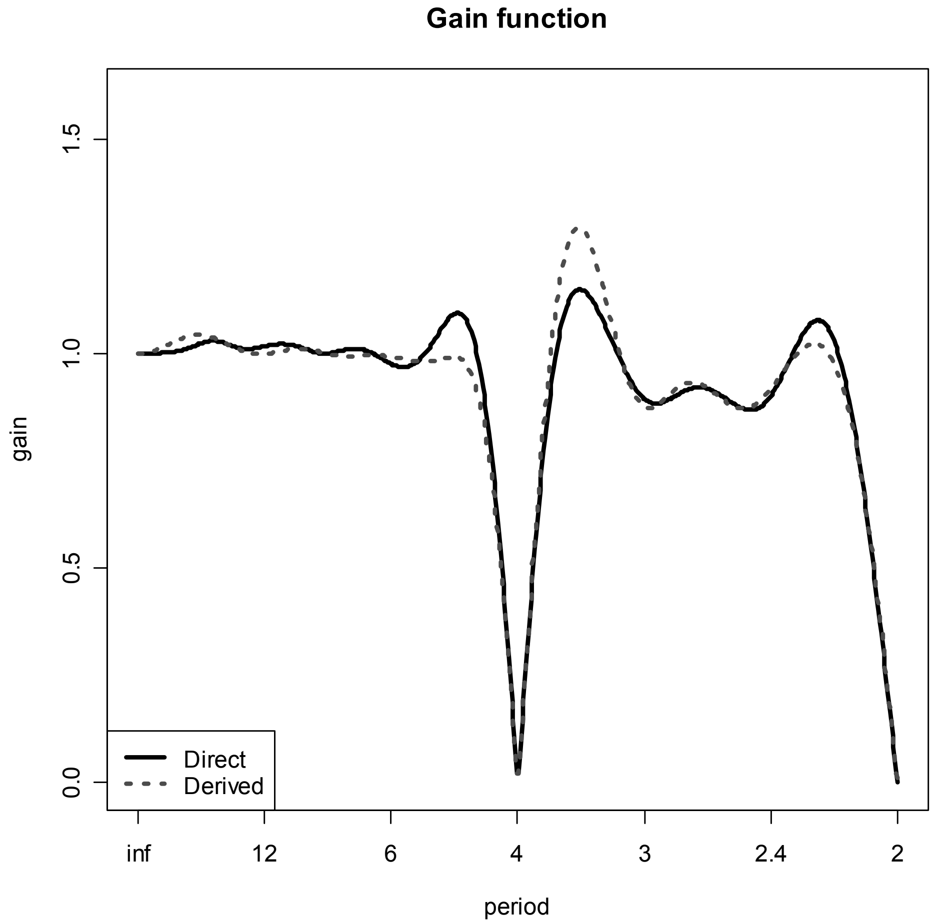
Figure 4.4 shows the second (lag 1) asymmetric seasonal adjustment weights of the two approaches. Their weights are similar but different for some lags.

4.4 The second (lag 1) asymmetric seasonal adjustment weights



To understand the impact on revisional behaviour of the second asymmetric weights under the two different seasonal adjustment approaches, we plot their gain functions in figure 4.5.

4.5 The gain function of the second (lag 1) asymmetric filter of the two approaches



The gain functions of the two approaches exhibit very similar frequency filtering properties except for the periods at about 3.5 and five quarters. The direct approach over-amplifies power at about the five quarter period compared to the temporally derived approach. In contrast, the temporally derived approach amplifies more power at about the 3.5 quarter period compared to the direct approach. In other words, better frequency filtering properties are displayed by the temporally derived approach at about the five quarter period and by the direct approach at about the 3.5 quarter period. Hence, it is very hard to judge which approach is superior from a frequency filtering perspective.

Figure 4.6 summarises the relative revisions against previous estimates for earlier observations by lags. This is indicated by plotting the derived relative revisions against those of the direct approach. The slopes of the mini-graphs in figure 4.6 reflect the relative revisions of the temporally derived approach to that of the direct approach. For example, the mini-graphs for lags 2, 3, 6, 7 and 9 indicate that there is not much difference between the temporally derived and direct approaches. Lags 1, 4, 5, 8, 10 and 11 indicate slightly larger revisions for the temporally derived approach than for the direct approach. The greatest revision difference, favouring the temporally derived approach, is indicated at lag 0. However, the revision at lag 0 is not observable in practice.

In general, the relative revision patterns of the two approaches are very close, but the revision sizes are in favour of neither the direct approach nor the temporally derived approach for all lags as a whole. This is also demonstrated by the relative revision patterns of two individual replicates presented in Appendix A.

4.6 Summary of revision simulation (1000 replicates): derived against direct for lags 0 to 11

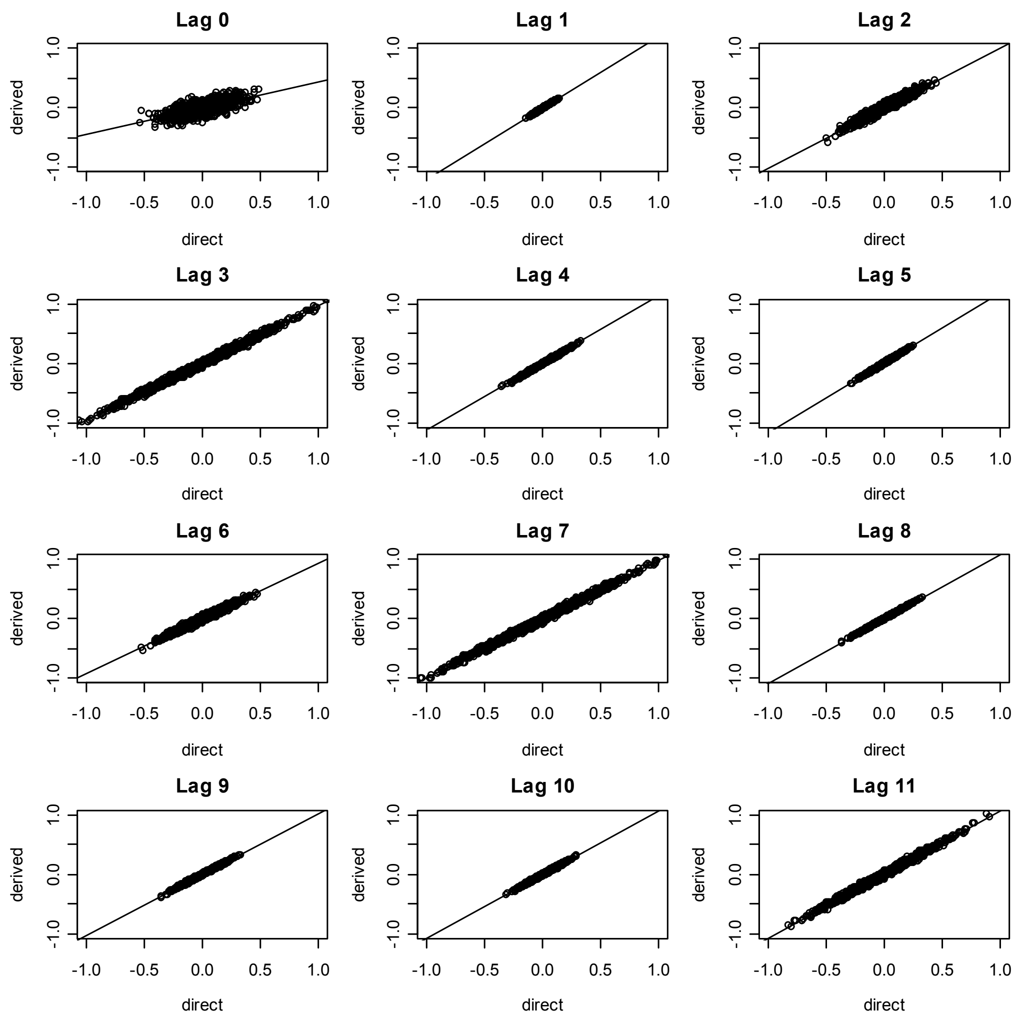
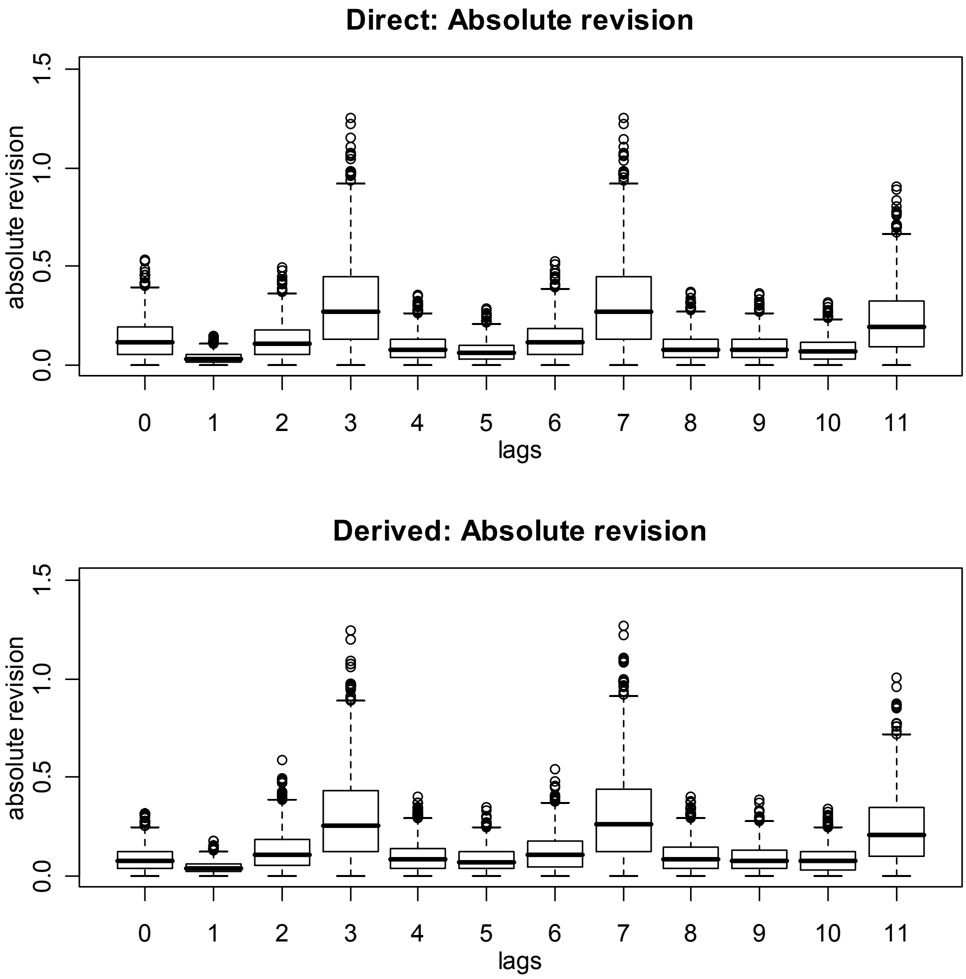


Figure 4.7 shows the variation of the absolute relative revision of the two approaches in box plots. Their relative revision patterns are similar. However, by comparing the corresponding medians and inter-quartile ranges, we can see that the revision sizes at lags 1, 4, 5, 8, 10 and 11 indicate slightly larger revisions for the temporally derived approach than for the direct approach. The greatest revision difference, favouring the derived approach, is indicated at lag 0. However, the revision at lag 0 is not usually observable in practice.

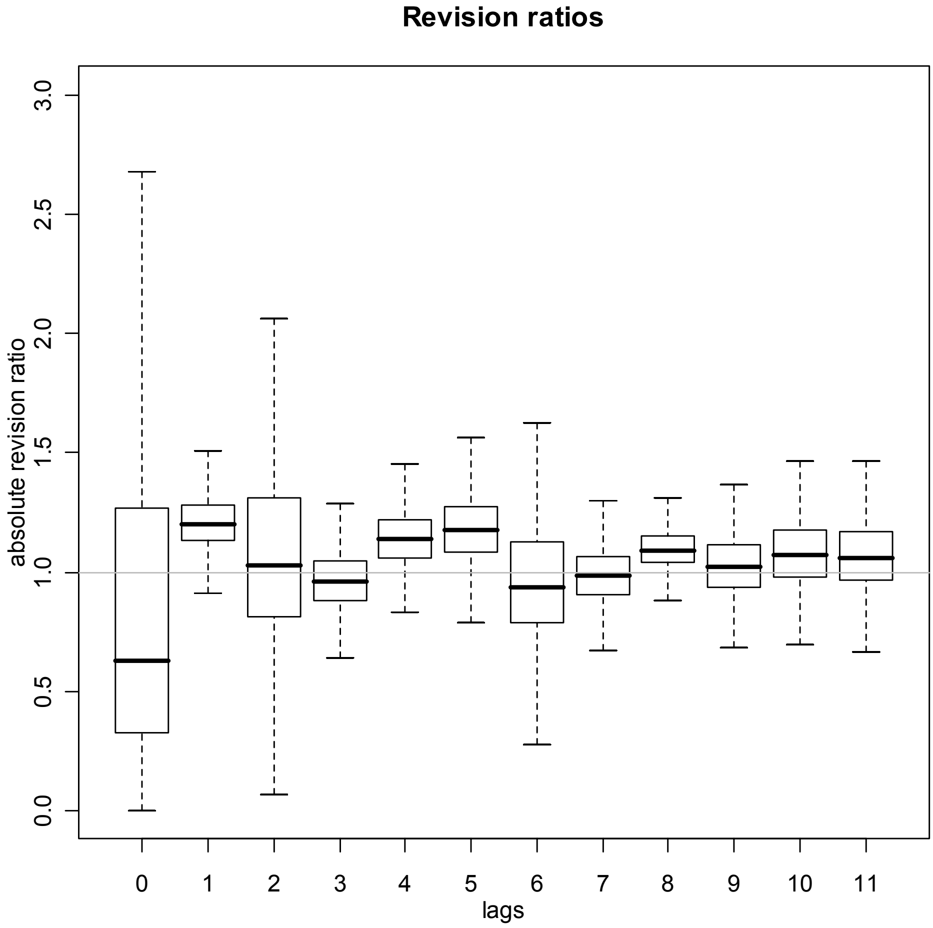
4.7 Boxplots (variation) of the absolute relative revision for the two approaches



The relative revision comparison of the two approaches at specific lags at the current end is summarised in figure 4.8 in term of their revision ratios (derived/direct) which can be interpreted as a revision efficiency measure of the temporally derived approach against the direct approach. Revision ratio box plots with bars below the unity line indicate relative revisions of the temporally derived approach are more efficient than those of the direct approach. Above the unity line the converse is true, that is, relative revisions of the direct approach are more efficient than those for the derived approach at that lag.

We can conclude that the two approaches have different relative revision properties driven by their different asymmetric filters. However, their relative revision sizes are not significantly distinguishable over all previous periods, although the temporally derived approach seems to have slightly larger relative revisions on lag 1, 4 and 5.

4.8 Revision ratios (derived/direct) of the absolute relative revision for the two approaches



This relative revision Monte-Carlo simulation study demonstrates that there are no clear differences between the direct and temporally derived approaches in terms of relative revision to previous estimates. However, this simulation study cannot conclude which approach is better in terms of their initial concurrent estimates against their stable estimates when more future observations become available.

Familiarity with the behaviour of current end relative revisions caused by the ordering of the application of seasonal and aggregation filters is important before embarking on the reliable revision concepts and its impact assessment measures described in the next section.

5. METHODS

Our methods are dual focussed. We wish to assess the revisional impact of the temporally derived approach for the ABS X11 method with and without the ARIMA forecasting extension, since this is not covered by the literature as far as we know. The quality of the seasonal adjustment estimates obtained by the two approaches is also compared since a reduction in the quality of seasonal adjustment due to a change in approach is not desirable.

The results presented in this paper were calculated based on synthetically simulated and real time series. Using synthetically simulated time series allows us to make a quality assessment from an accuracy perspective since a 'true' quarterly seasonally adjusted benchmark is known. For real data, a 'true' quarterly seasonally adjusted benchmark is not known. An approximate 'stable' quarterly seasonally adjusted is adopted for revisional assessments. These terms are defined in the following sections.

5.1 Simulated data and the 'true' benchmark

Twenty replicates[1] of monthly time series, y_t, were simulated using the model (5.1) (Lee, 1982) which is a well known model for evaluating seasonal adjustment methods in the literature (see Cleveland, 1980):

$$y_t = 100t + t \times \cos\left[\frac{2\pi(t-0.5)}{12}\right] + 60e_t \tag{5.1}$$

where time $t = 1, 2, ..., 240$ and e_t is the error term following an *i.i.d.* N(0,1) distribution. The first term is regarded as the trend component of the model, the second term the seasonal component and the third term the noise (or irregular) component. Thus the 'true' benchmark seasonally adjusted quarterly series is easily determined, that is, the temporal aggregate of y_t minus the temporal aggregate of the seasonal component.

$$y_t - t \times \cos\left[\frac{2\pi(t-0.5)}{12}\right]$$

The start date adopted for these series was January 1980. These 20 original series were then temporally aggregated to obtain 20 quarterly original equivalent time series. The 20 quarterly true benchmark series were also determined.

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1 Only twenty replicates are produced for this Monte-Carlo simulation because the simulated series is reasonably stable. The revision analysis is more sensitive to the length of the time series than to the number of replicates. In addition, the revision simulation is computationally intensive for a large number of replicates.

5.2 Real data

A total of nine ABS Balance of Payments goods debits and credits time series, were used for real data analysis. These consisted of the monthly and quarterly original equivalent time series for nine general merchandise Balance of Payments time series, three from the rural goods credits side of the accounts:

1. meat and meat preparations;
2. wool and sheepskins; and
3. other rural;

and six from the consumption goods (debits side) of the accounts:

1. food and beverage, mainly for consumption;
2. household electrical items;
3. non-industrial transport equipment;
4. textiles, clothing and footwear;
5. toys, books and leisure goods; and
6. consumption goods n.e.s. (nowhere else specified).

See ABS (2008a) for monthly data and ABS (2008b) for quarterly data.

We mentioned the use of ‘clean’ original data in the introduction. To recap, we believe, based on literature and our earlier research (see Zhang *et al.*, 2005), that calendar-related effects and outliers are better estimated from monthly series rather than their quarterly counterparts. Calendar effects, such as trading day, Easter, for example, were hence corrected before X11 decomposition for the monthly series. The prior corrected quarterly series were then obtained as temporal aggregates of their equivalent prior corrected (clean) monthly series. To evaluate the revision performance of the direct and derived approaches, we used these clean monthly and quarterly series as our ‘original’ series.

5.3 Revision simulations and the stable estimates

Revision simulations are generally run on sub-spans of a time series with the sub-spans chosen so that there will be enough prior and subsequent data points available to generate a stable estimate. The stable estimate usually requires three years of additional data to reach, i.e. for a particular time t, after more than three years of additional data become available, the seasonally adjusted estimate for time t is stable. The revision against the stable estimate can be interpreted as the size of revision required for an estimate to reach the stable estimate.

Lagged estimates are obtained by using available data at successive time points. For example, a lag zero December 1999 estimate would be the initial estimate using data up to December 1999 while the lag one estimate would be the second estimate of December 1999 using data up to January 2000. The concurrent (or ARIMA forecast) estimate is revised to incorporate new information as more data become available. Thus for each time point in the simulation span, several estimates are calculated at different time lags, reflecting the different levels of data available for calculation of the concurrent estimate. Similarly, several ARIMA forecast estimates are calculated at different time lags.

Revision simulations were conducted on the 20 replicates of the synthetically simulated monthly and quarterly original series of 17 years (assuming from January 1980 to December 1996) based on an additive decomposition model without any prior corrections for both the concurrent seasonal adjustment method and ARIMA forecast extension method. Furthermore, standard X11 settings, that is 3×5 seasonal and 13-term Henderson trend filters for monthly series, and 3×5 seasonal and 5-term Henderson trend filters for quarterly series, were applied in practice.

For the real data, revision simulations based on a multiplicative decomposition model were conducted on both the nine monthly Balance of Payments series with no prior corrections and their quarterly equivalents over a data span from July 1981 to March 2008. Again standard X11 settings were applied in practice, that is, 3×5 seasonal and 13-term Henderson trend filters for monthly series, and 3×5 seasonal and 5-term Henderson trend filters for quarterly series.

The ARIMA forecasting extensions were based on the Airline model (Box and Jenkins, 1970) which is adequate for both the simulated and real time series. However, the parameters of this model are re-estimated each year within the simulation span to emulate, to some extent, ABS annual seasonal reanalysis procedures.

Revision simulations were used to assess revision performance as described in the next section.

5.4 Revision performance measure

The results from the simulations outlined in Section 5.3 were used to compare the revision performance of direct seasonal adjustment with the temporally derived approach. Revisional impact was assessed by comparing the seasonally adjusted estimates obtained from the concurrent X11 method, with and without the ARIMA forecasting extension (for both direct and derived approaches), to their equivalent stable estimates. The measure used to assess revision performance was the mean absolute percentage revision for both level and movements (percentage of first difference) as described below.

The percentage revision of level and movement for each estimate, are defined in equations (5.2) and (5.3) following, respectively:

$$R^X_{t|t+k} = 100\frac{(\hat{X}_{t|t+k} - X_{t|t+K})}{X_{t|t+K}} \tag{5.2}$$

$$\Re^X_{t|t+k} = 100\left(\frac{(\hat{X}_{t|t+k} - \hat{X}_{t-1|t+k})}{X_{t-1|t+k}} - \frac{(\hat{X}_{t|t+K} - \hat{X}_{t-1|t+K})}{X_{t-1|t+K}}\right) \tag{5.3}$$

where $\hat{X}_{t|t+k}$ is the estimate of the quantity of interest, such as level of the seasonally adjusted series, for time period t, made using all the information available up to time period $t+k$, and $\hat{X}_{t|t+K}$ is the corresponding stable estimate made using K periods of additional data (usually more than three years).

The mean absolute percentage revision and the mean absolute percentage movement revision defined in equations (5.4) and (5.5) following are a summarised statistical measure for the size of the revisions at a particular lag k;

$$\overline{R^X_k} = \frac{1}{n_k}\sum_{t=s}^{e}\left|R^X_{t|t+k}\right| \tag{5.4}$$

$$\overline{\Re^X_k} = \frac{1}{n_k}\sum_{t=s}^{e}\left|\Re^X_{t|t+k}\right| \tag{5.5}$$

where time $t=e$ is the end point of the simulation, $t=s$ is the start point of the simulation and n_k is the number of observations available in the simulation span at lag k. In other words, it is the average of the absolute revision between the estimates, produced after k periods and the stable estimates over the simulation span.

The mean absolute percentage revision, $\overline{R_k^X}$, and the mean absolute percentage movement revision $\overline{\Re_k^X}$ were calculated for each lag k using both direct and temporally derived quarterly seasonally adjusted outputs for the four scenarios:

1. concurrent method level estimates;
2. concurrent method movement estimates;
3. ARIMA forecast method level estimates; and
4. ARIMA forecast method movement estimates.

5.5 Seasonal adjustment quality measures

The following five quality measures form a basis for evaluation of direct seasonal adjustment against the temporally derived approach and were calculated for all quarterly seasonally adjusted series.

1. residual seasonality probability;
2. mean square error (applicable to synthetically simulated data only);
3. average absolute percentage change;
4. first measure of smoothness R1; and
5. second measure of smoothness R2.

1. Residual seasonality

The residual seasonality of the quarterly seasonally adjusted estimates was assessed using a simple ANOVA on the residuals (of seasonally adjusted and trend estimates) and resultant F-test probability. The residuals were obtained by first producing Henderson trend estimates from the seasonally adjusted estimates (direct and temporally derived) based on ABS publication standards of a 7 term Henderson filter and I/C ratio (the asymmetric Henderson filter end weight) of 1. The trend estimates were then subtracted from their seasonally adjusted series to obtain the residuals.

2. Mean square error (MSE)

The accuracy of resultant quarterly seasonally adjusted estimates was assessed by calculating their *MSE* against their 'true' benchmark quarterly seasonally adjusted series. The *MSE* was calculated using formula (5.6) following:

$$MSE_{method} = \left[\sum_{t=1}^{n} (SA_{t,method} - SA_{t,bench})^2 \right] \Big/ n \tag{5.6}$$

where $t = 1, 2, ..., n$; n = number of data points (68 for our simulations); $SA_{t,method}$ = the quarterly seasonally adjusted estimate at time t for approaches of direct or temporally derived; and $SA_{t,bench}$ = the quarterly seasonally adjusted true benchmark at time t.

3. Average absolute percentage change (AAPC)

The average absolute percentage change of a seasonally adjusted series provides a measure of its volatility. It considers the size of the period-to-period movements throughout the series, irrespective of sign, and averages these to obtain the expected size of movements for this series. Hence the AAPC was calculated using equation (5.7):

$$AAPC = \frac{1}{n-1} \sum_{t=2}^{n} \left| \frac{SA_t - SA_{t-1}}{SA_{t-1}} \right| \tag{5.7}$$

4. Smoothness measure R1

It is a desirable property of any seasonal adjustment procedure that the seasonally adjusted estimates produced are smooth. R1 denotes the mean of the squares of the first differences of the seasonally adjusted series and is calculated using the following equation (5.8). The lower the R1 value, the 'better' the seasonal adjustment.

$$\mathrm{R1} = \frac{1}{n-1} \sum_{t=2}^{n} (SA_t - SA_{t-1})^2 \tag{5.8}$$

5. Smoothness measure R2

R2 denotes the mean of the squares of the additive residual-irregulars, that is the seasonally adjusted series minus the trend. Again the lower the R2 value, the smoother the seasonal adjustment. Algebraically, R2 is calculated using equation (5.9), where SA_t are still the quarterly seasonally adjusted estimates at time t, and T_t are the corresponding trend estimates at time t.

$$\mathrm{R2} = \frac{1}{n} \sum_{t=1}^{n} (SA_t - T_t)^2 \tag{5.9}$$

5.6 Relative measures for comparison of temporally derived and direct seasonal adjustments

The revision performance of direct seasonal adjustment and the temporally derived approach was assessed by mean absolute revision performance, leading to the following measures:

1. the average mean absolute percentage revision of individual lags over the 20 replicates; and
2. the average mean absolute percentage revision ratio of temporally derived to direct. This is a quality measure to represent the efficiency of the temporally derived approach. For example, an average mean absolute percentage revision ratio less than unity suggests that the derived approach is more efficient in revision performance than the direct approach.

The quality of the quarterly seasonally adjusted series obtained via direct seasonal adjustment or the temporally derived approach was evaluated using the following combinations of quality and efficiency measures:

1. their residual seasonality probabilities;
2. $MSE_{derived} / MSE_{direct}$;
3. $AAPC_{derived} / AAPC_{direct}$;
4. $\mathrm{R1}_{derived} / \mathrm{R1}_{direct}$; and
5. $\mathrm{R2}_{derived} / \mathrm{R2}_{direct}$.

6. RESULTS

6.1 Monte-Carlo simulation

6.1.1 Revision performance assessment

The average (over 20 replicates) of mean absolute percentage revision graph presents the mean absolute percentage revision calculated from the experience of the series. These graphs show the average revision at particular lags of interest and can be used to indicate both the average size of revision required to reach the stable estimates and the rate of convergence.

Figure 6.1 compares the average of mean absolute percentage revisions over 20 replicates for the seasonally adjusted estimates of the temporally derived and direct quarterly under the following four scenarios:

1. concurrent method level estimates;
2. ARIMA forecast method level estimates;
3. concurrent method movement estimates; and
4. ARIMA forecast method movement estimates.

In figure 6.1 we can see that for seasonally adjusted level estimates, the revision performance of the temporally derived approach is better than that of direct adjustment. This holds for both the ABS concurrent X11 method and the ABS X11 method with the ARIMA forecasting extension. Furthermore, the gain in revision reliability is greater when the ARIMA forecasting extension is used. Similar conclusions can be drawn by comparing the period-to-period movement graphs.

We can also see that seasonal adjustment with ARIMA forecast extension has smaller revisions than the concurrent method for both direct and derived approaches respectively. This feature is attributed to the better forecast performance of ARIMA over the X11 implied forecasts.

6.1 Comparison of average mean absolute percentage revisions direct and temporally derived seasonally adjusted quarterly series

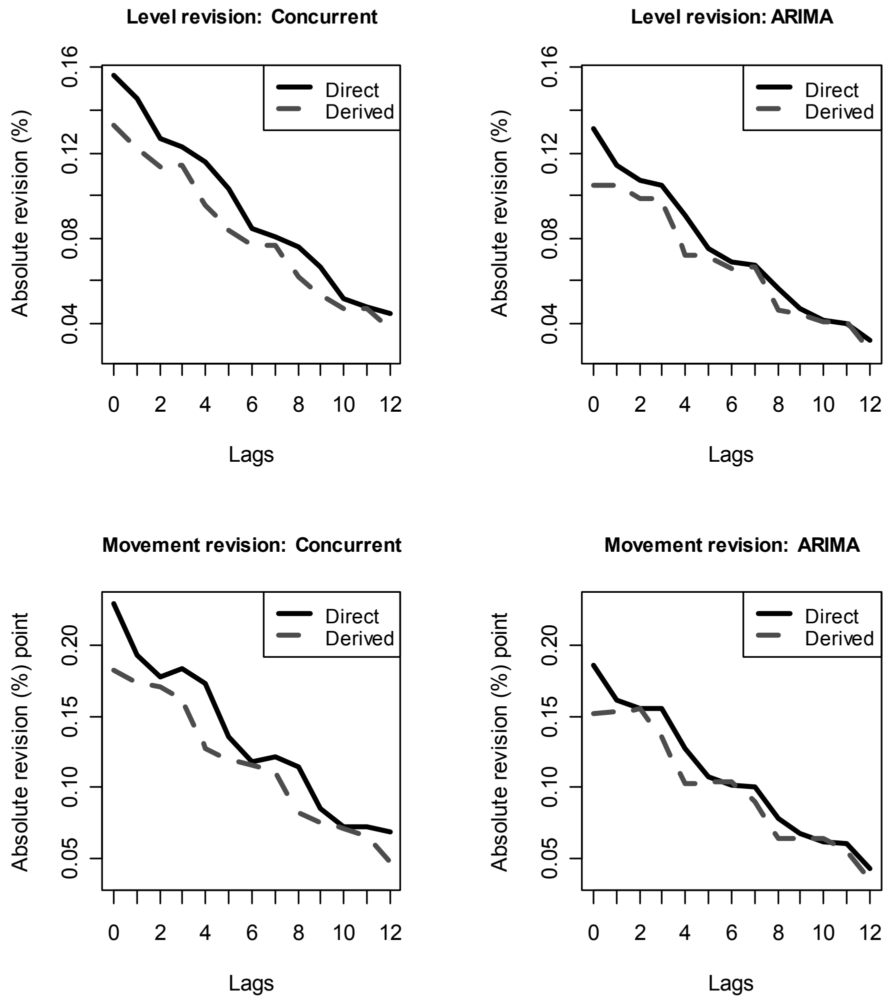
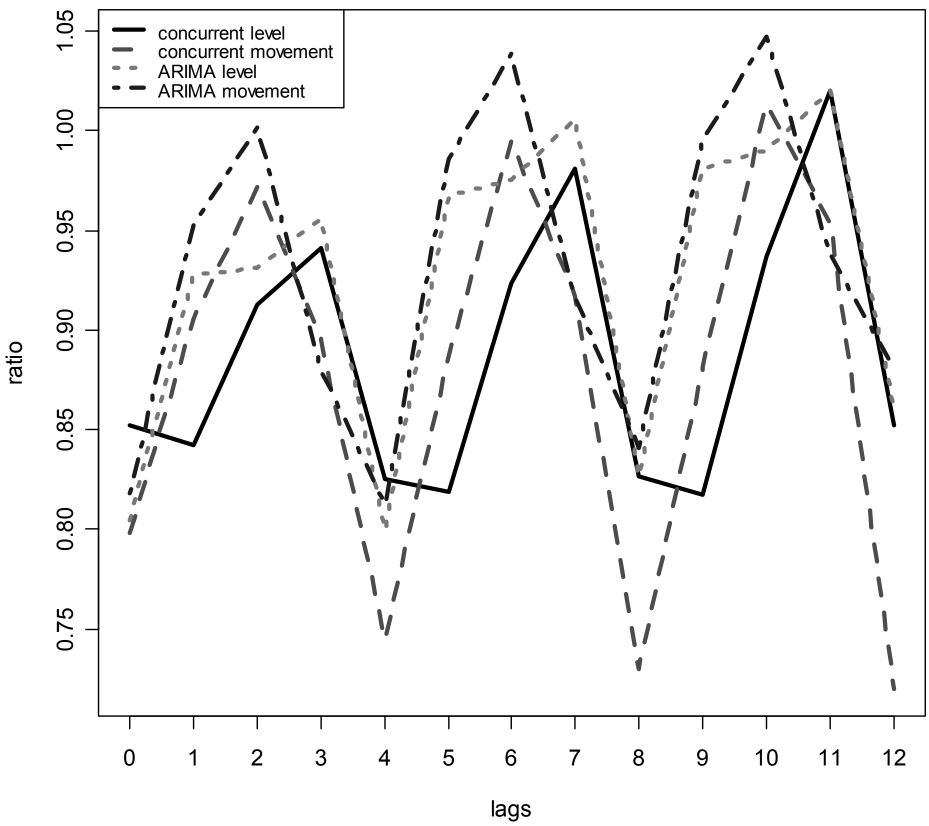


Figure 6.2 depicts the average of the mean absolute revision ratio of the seasonally adjusted estimates from the temporally derived and direct approaches. Values below unity indicate that, on average, the temporally derived approach is performing better in revision efficiency.

For both level and movement estimates, we can see that, for the concurrent X11 seasonal adjustment method, the temporally derived approach outperforms revisionally for all lags except lag 11. For the ABS X11 method with ARIMA forecasting, the same comment holds except for lags 7 and 11 in this instance. The greatest revisional gains of the temporally derived approach, for both with and without ARIMA forecasting methods, are made in the more recent estimates (first few lags).

The ratios also show a systematic seasonal pattern. The temporally derived approach gains more efficiency every four lags. This seasonal pattern indicates that the X11 seasonal factor update for monthly series may be more efficient than for the quarterlies when asymmetric seasonal filters are updated annually towards to the symmetric seasonal filter.

6.2 Average revision ratio for temporally aggregated seasonally adjusted series to directly seasonally adjusted series



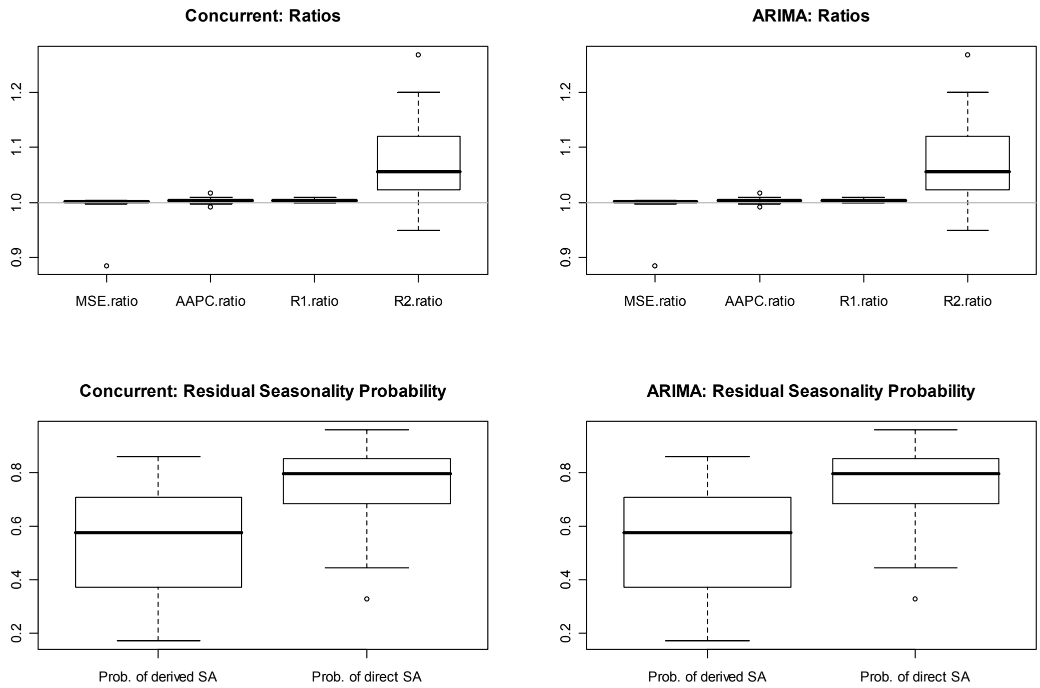
In summary, for this simulation study, revision performance is improved by the temporally derived approach for both the ABS X11 concurrent method and the ABS ARIMA forecasting extension. This revision improvement implies that the forecasting performance by predicting monthly series and then aggregating into quarterly frequency is more accurate than forecasting the quarterly series directly. This is consistent with Wei's (1978) forecast accuracy claim and is reflected in the revision behaviours of the temporally derived and direct approaches.

6.1.2 Seasonal adjustment quality comparison

Although the main focus of this paper is on revisional performance, ensuring there is no loss in seasonal adjustment quality due to the temporally derived approach is also important.

For both the concurrent ABS X11 method and the ABS X11 with the ARIMA forecasting extension, figure 6.3 provides a comparison of the quality measure ratios of MSE, AAPC, R1 and R2 calculated from the temporally derived to direct approaches as presented in Sections 5.5 and 5.6 along with the residual seasonality probabilities.

6.3 Boxplots of quality measure ratios and residual seasonality probabilities



The top two box plots of figure 6.3 show not much difference between the temporally derived and direct approaches for both the concurrent seasonal adjustment method and ARIMA forecast extension method except for the R2 ratio. The R2 ratio is larger than 1. This indicates that the derived approach is less smooth than the direct approach.

The bottom two box plots of figure 6.3 compare the residual seasonality probability for the seasonally adjusted estimates from both the temporally derived and direct approaches. They show no statistical evidence suggesting that there is any seasonality left in the seasonally adjusted estimates from the temporally derived approach.

The overall result from the Monte-Carlo simulation exercise is that the temporally derived approach does not lead to an unacceptable loss in the quality of seasonal adjustment when compared to the direct adjustment.

6.2 Real data analysis (Balance of Payments)

6.2.1 Revision performance assessment

Graphical summaries of the revisional performance of all nine Balance of Payments series can be found in Appendix B. We highlight and discuss two examples here for illustrative purposes.

Figure 6.5 depicts the revision performance of non-industrial transport equipment. The shadowed area is the standard error range of the mean absolute revisions of the direct approach. It can be seen that for this series, the temporally derived approach is better than the direct approach for both the concurrent ABS X11 method and the ARIMA forecast extension approach. This figure also shows how the ARIMA forecast method provides lower revision regardless of the adjustment (temporal vs direct) approach for both level and movements.

6.5 Non-industrial transport equipment: Comparison of average percentage revisions

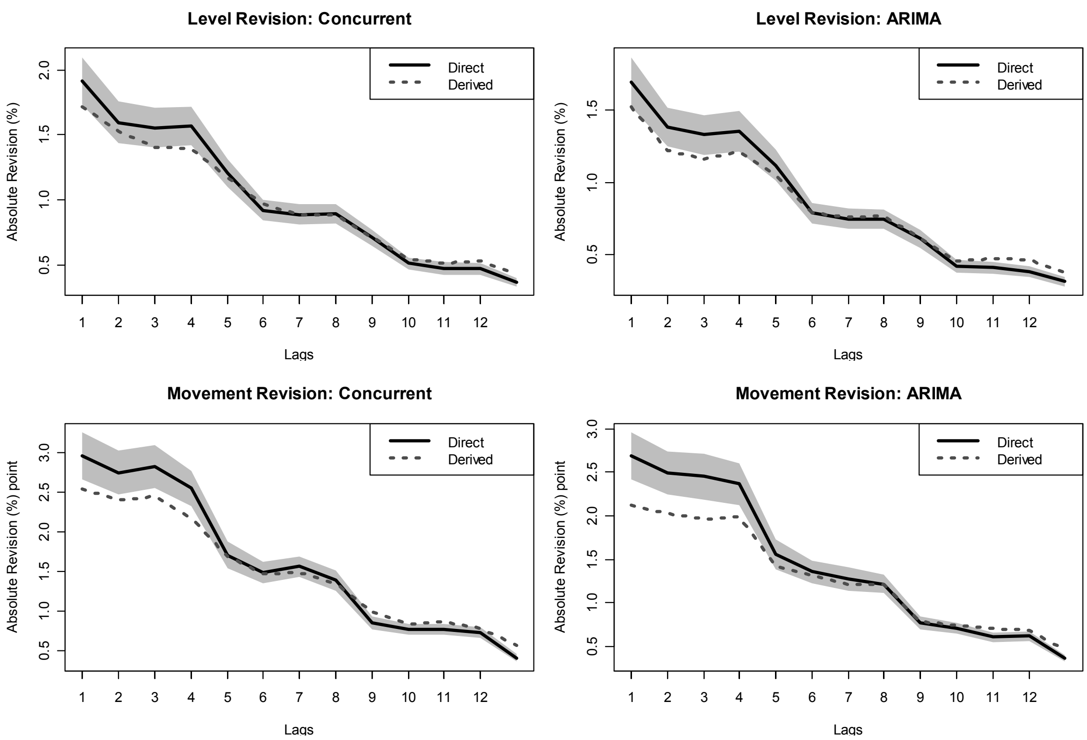
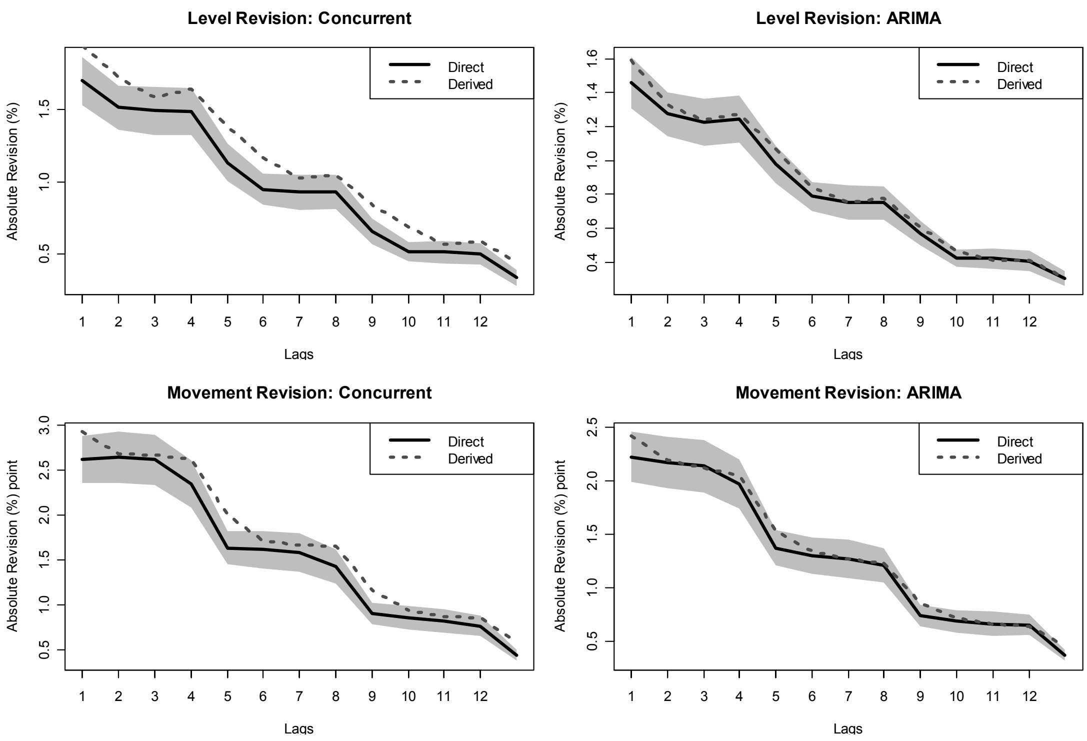


Figure 6.6 depicts the revision performance of textiles, clothing and footwear. It can be seen that for this series, the comparison between the temporally derived approach and direct adjustment does not yield conclusive results. For level revisions, the temporally derived approach is arguably inferior to direct adjustment for the concurrent ABS X11 method. For the ARIMA forecast extension approach, level revisions are very similar for the two approaches. The movements tell a similar story. This figure again shows that the ARIMA forecast method provides overall lower revision regardless of the adjustment (temporal vs direct) approach for both level and movements.

6.6 Textiles, clothing and footwear: comparison of average absolute percentage revisions



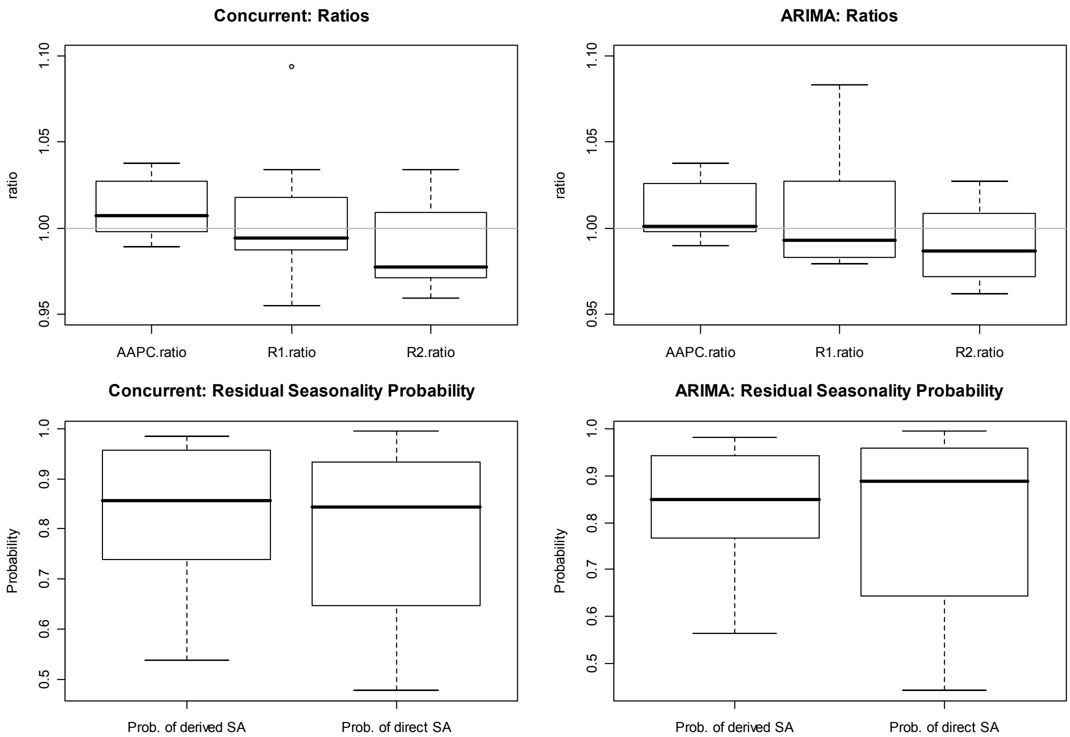
It can be seen that in general, these results are non-conclusive for revision performance of the two approaches. However, they consistently show that using the ARIMA forecast extension produces smaller revisions than the concurrent method regardless of whether the adjustment approach is direct or temporally derived.

6.2.2 Seasonal adjustment quality comparison

Figure 6.7 provides a comparison of the quality measure ratios of AAPC, R1 and R2 from the seasonally adjusted estimates of the temporally derived and direct approaches as presented/discussed in Sections 5.5 and 5.6 along with the residual seasonality probabilities for both the concurrent X11 method and the ARIMA forecasting extension.

Comparing the two pairs of box plots, it can be seen that from a quality perspective there is little difference between the direct and temporally derived approaches.

6.7 Boxplot of quality measure ratios and residual seasonality F-test probabilities: Concurrent method and ARIMA forecast method



The overall results from the real data analysis suggest that the temporally derived approach provides equally good quality of seasonally adjusted estimates when compared to the direct approach.

7. CONCLUDING REMARKS

Users of official statistics can choose the original, seasonally adjusted or trend estimates either individually or in combination to aid in the decision making process. When equivalent time series pairs of original monthly and quarterly estimates are seasonally adjusted directly and published, the consistency between the seasonally adjusted estimates for the two frequencies often causes some problems for official statistical agencies and users alike.

This paper presents a study comparing an alternative approach to producing quarterly seasonally adjusted estimates when equivalent monthly time series are available (the temporally derived approach) to the current practice of direct seasonal adjustment. It is found for synthetically simulated data that the temporally derived approach is robust and does not compromise most seasonal adjustment quality measures. For revision performance there is a potential increase in the volatility of the initial (concurrent) seasonally adjusted estimates which will be revised when more future data become available. Importantly, due to the nature of the temporally derived approach, consistency between monthly and quarterly seasonally adjusted equivalent estimates is ensured, along with seasonally adjusted additivity. Real data analysis yielded inconclusive results for revision performance but supported remaining claims.

Our study in this paper focuses only on flow time series and one common pair of X11 parameter settings for monthly and quarterly time series. Other settings for different time series volatility characteristics have not yet been explored. However, we believe that the overall conclusions would be the same when an appropriate pair of X11 parameter settings are selected suitable for the characteristics of the monthly and quarterly time series. Our study could be enhanced and broadened by: (1) increasing the number of replicate series; (2) extending simulation variations of seasonal decomposition models to include true multiplicative and pseudo-additive models; (3) examining the effects of the temporally derived approach on a contemporaneous aggregation structure including a a high level aggregate time series; (4) examining the effect of sampling error on the temporally derived approach; and (5) exploring the temporal aggregation approach for stock time series.

This has been an initial study primarily aimed at comparing the revision performance of the temporally derived approach against direct seasonal adjustment for quarterly flow series. Our results potentially encourage us to change the current ABS seasonal adjustment practice to improve consistency of the ABS seasonal adjustment products and to further improve our own productivity. We recognise that seasonal adjustment is only one aspect of introducing the temporally derived approach as an ABS practice. There are some other aspects of official statistical products, closely related to seasonal adjustment, which need to be addressed.

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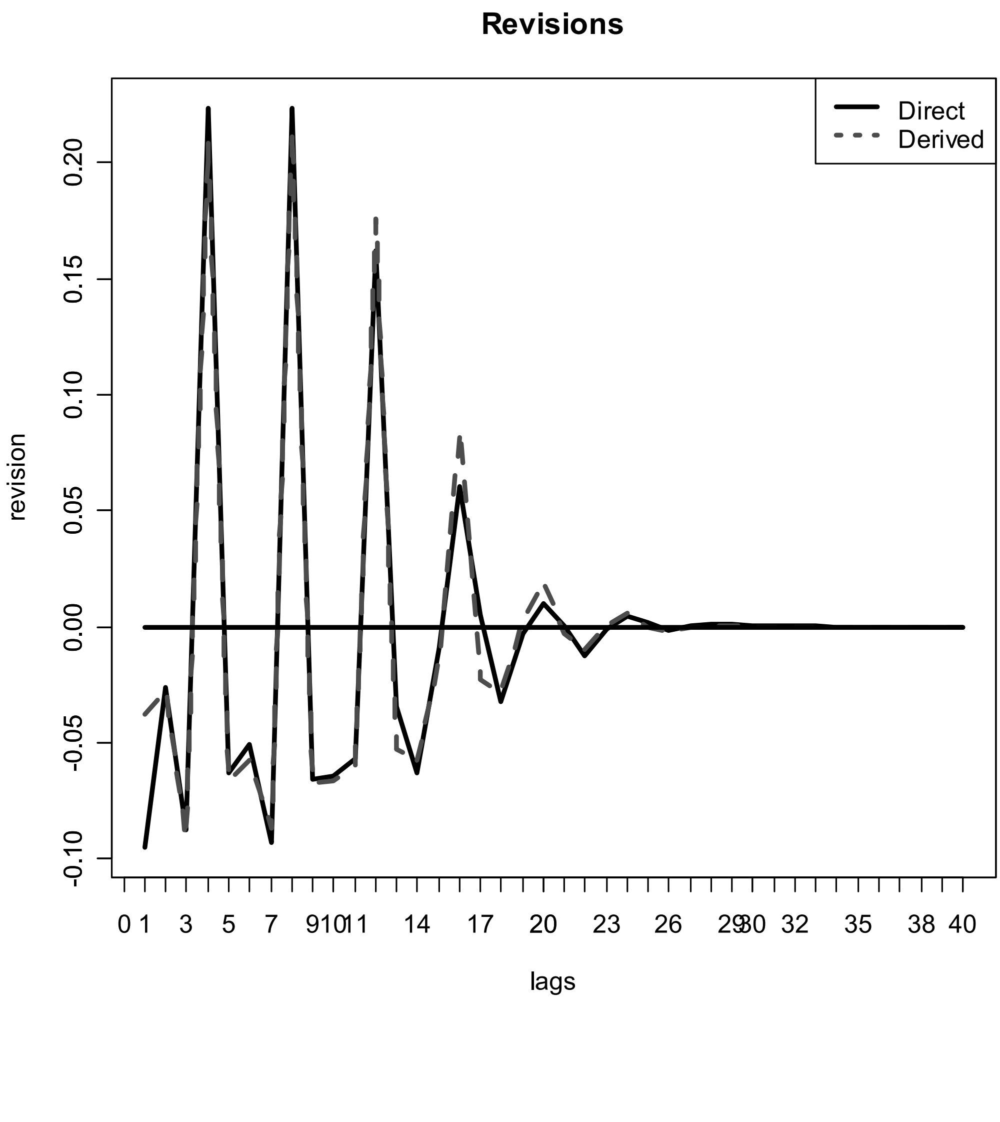
APPENDIXES

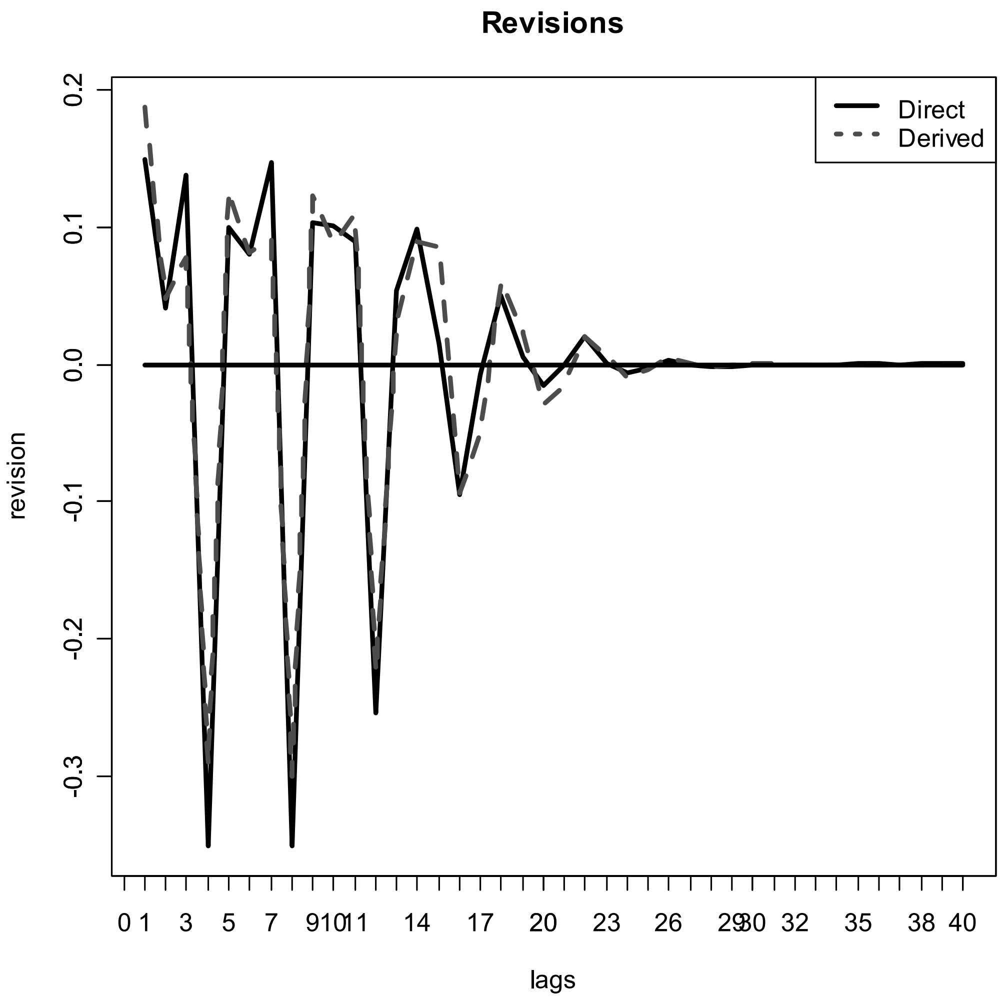
A. TWO CASES OF RELATIVE REVISION SIMULATIONS

Figures A.1 and A.2 display the revisions of two individual cases from the Monte-Carlo simulation exercise. These two cases show that the revision patterns of the two approaches are very close, but that the revision sizes favour neither the direct approach nor the temporally derived approach at all lags.

A.1 Revision pattern of case 1



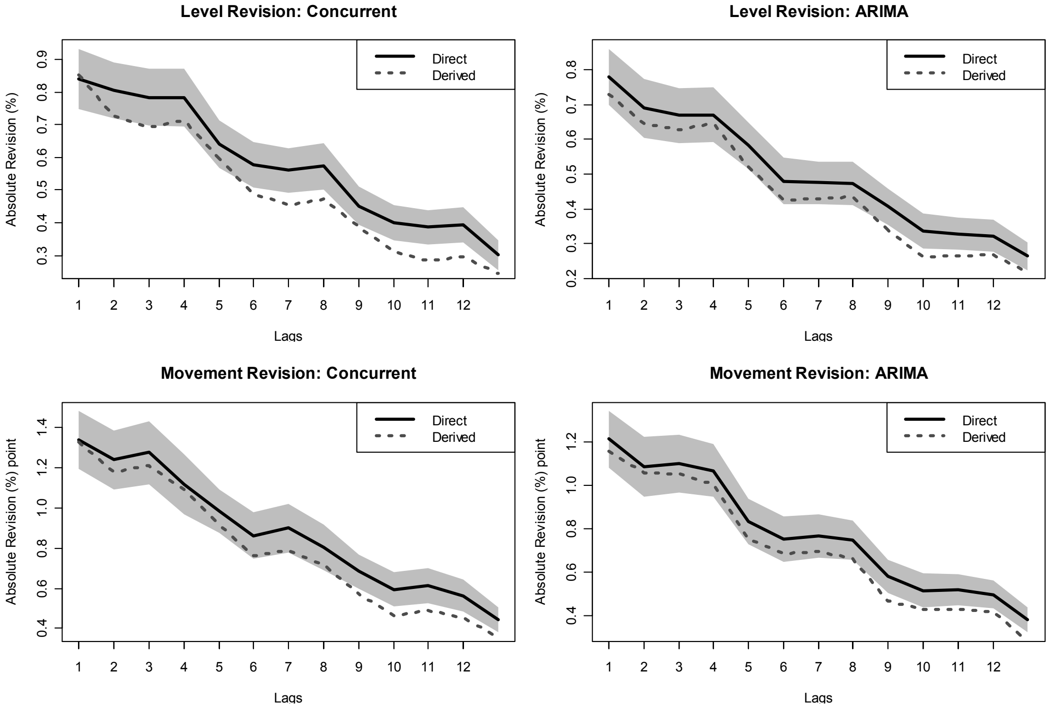
A.2 Revision pattern of case 2



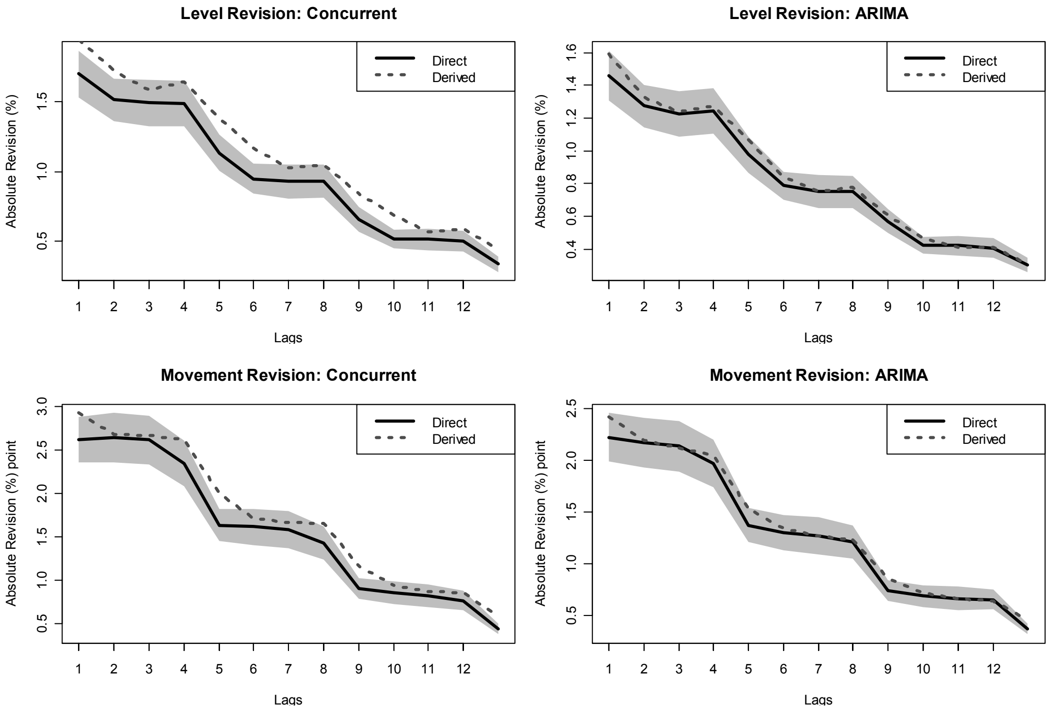
This relative revision Monte-Carlo simulation study demonstrates that there are no clear differences between the direct and temporally derived approaches in terms of relative revisions to the previous estimates. However, this simulation study cannot conclude which approach is better in term of their initial concurrent estimates against their stable estimates, when more observations become available in the future.

B. REVISION PERFORMANCE GRAPHS – REAL DATA (BALANCE OF PAYMENTS)

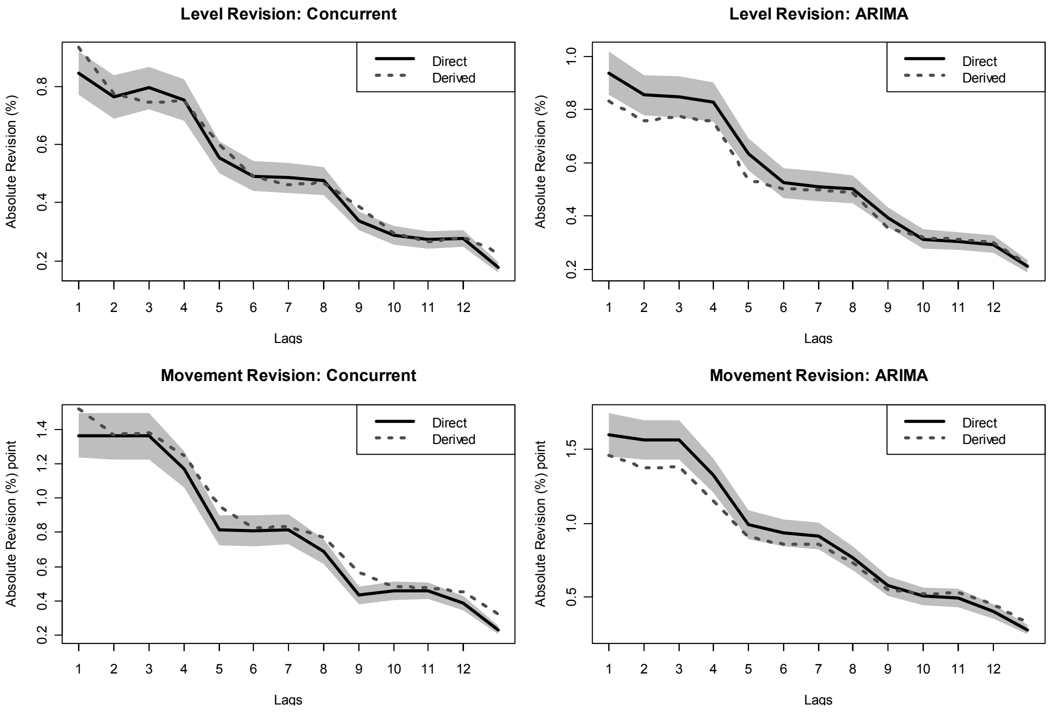
B.1 Consumption goods (n.e.s.)



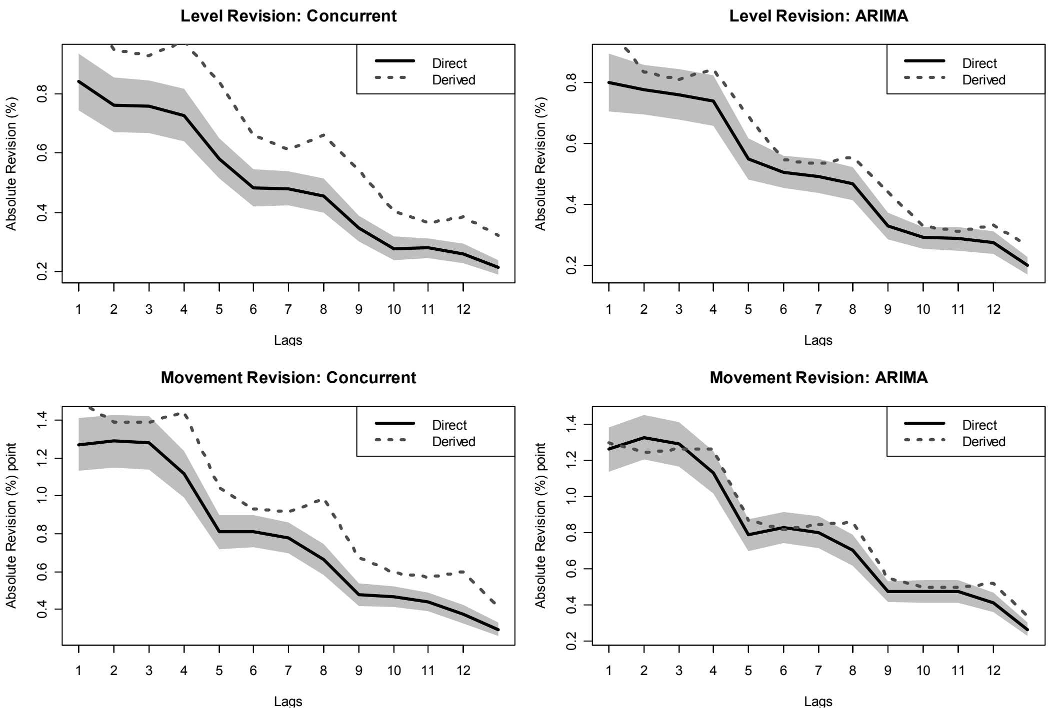
B.2 Textiles, clothing and footwear



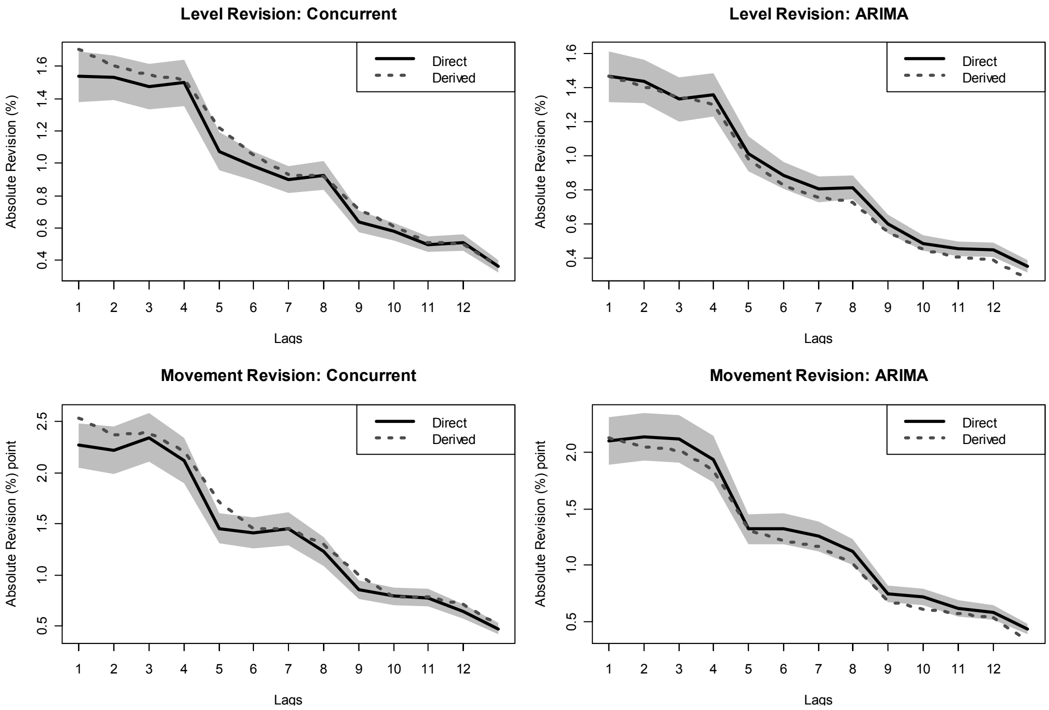
B.3 Toys, books and leisure goods



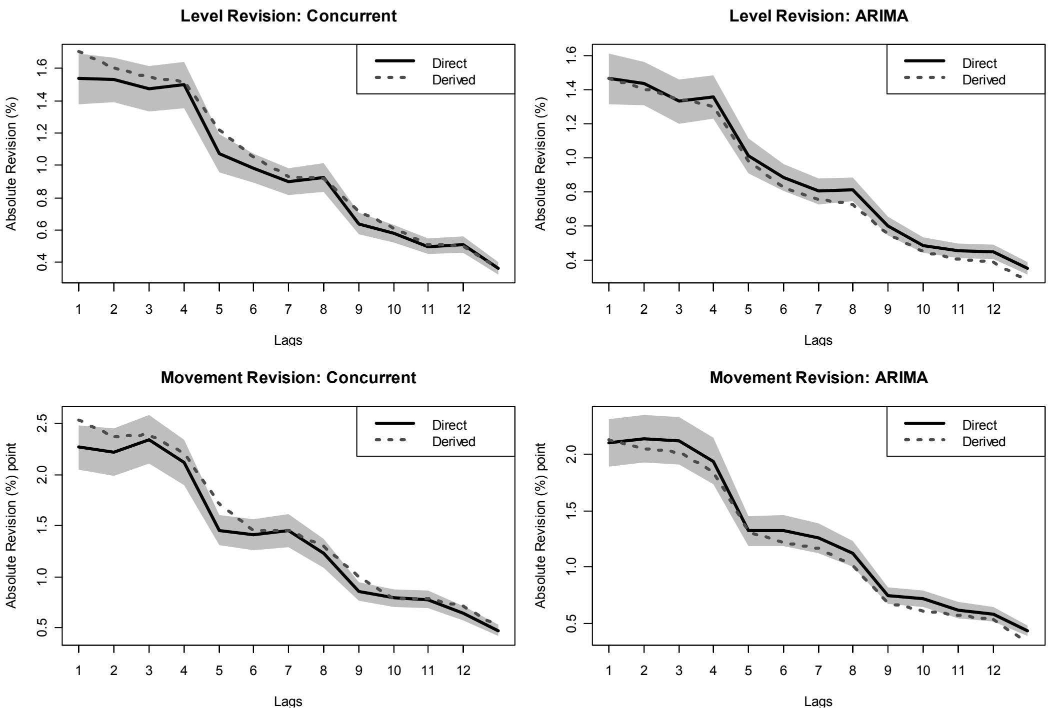
B.4 Food and beverages, mainly for consumption



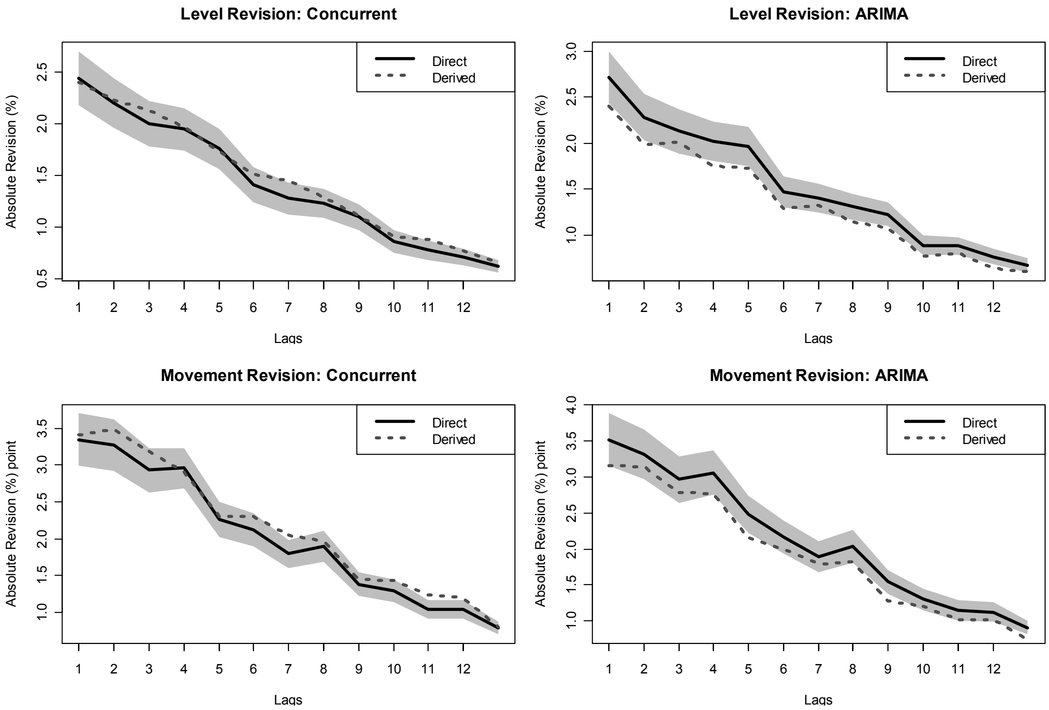
B.5 Household electrical items



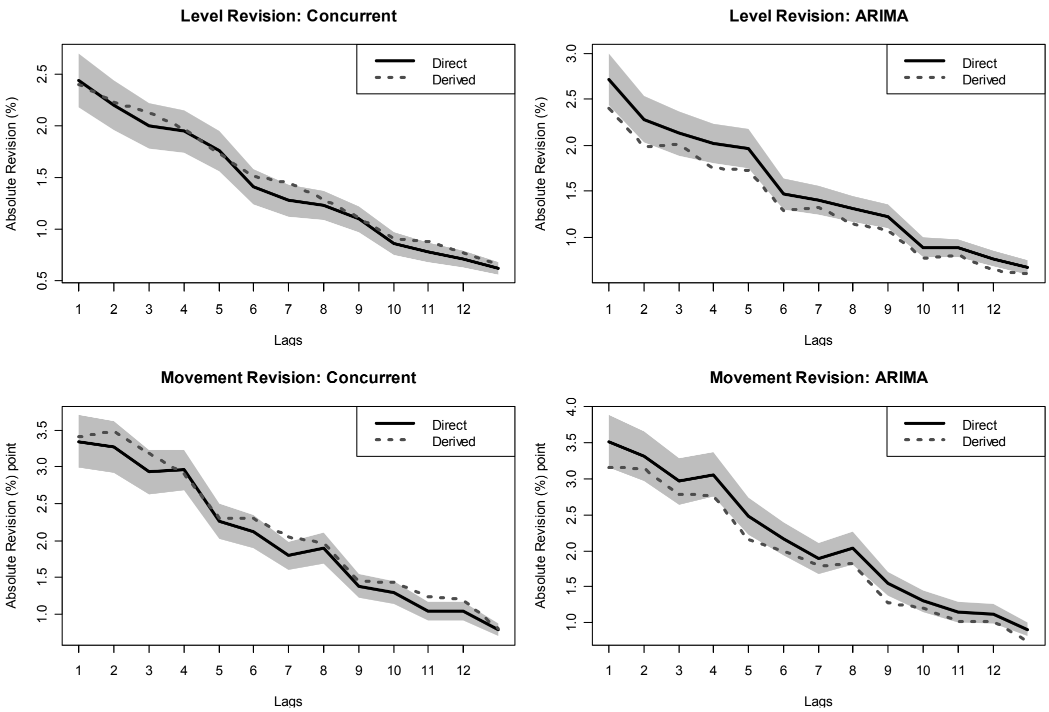
B.6 Non-industrial transport equipment



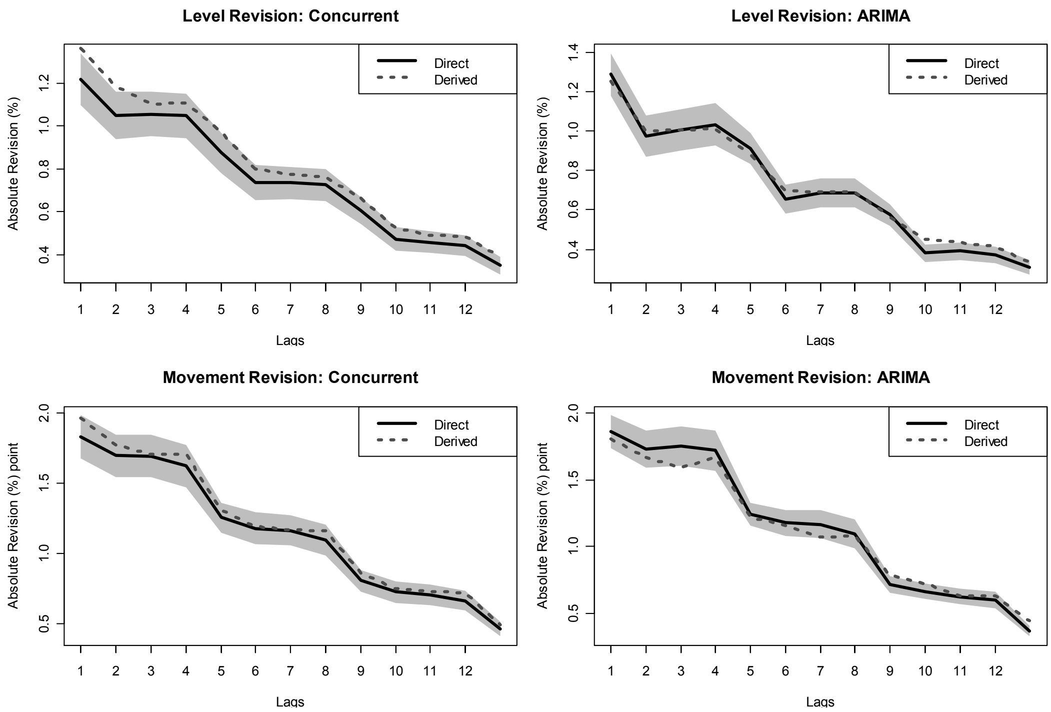
B.7 Other rural



B.8 Wool and sheepskins



B.9 Meat and meat preparations



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